PGroupN background theory

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Estimation of the deformations and load distributions in a group of piles generally requires the use of computer-based methods of analysis. Numerical techniques for pile group analysis may be broadly classified into two categories:

(1) load-transfer (or subgrade reaction) approaches;
(2) continuum-based approaches.

Programs based on the load-transfer or subgrade-reaction method

The above category is based on the so-called Winkler idealisation of the soil, i.e. the piles are modelled as a series of independent springs (e.g. the *t-z* or *p-y* curve methods). The main drawback to this approach is that it is based on empirical parameters (i.e. the modulus of subgrade reaction) which can only be backfigured from the results of pile load tests. However, in many practical situations, it is not possible to carry out such testing, at least in the preliminary stages of design. In addition, disregard of continuity through the soil oversimplifies the problem and makes it impossible to find a rational way to quantify the interaction effects between piles in a group. The computer codes GROUP (Reese *et al*., 2000) and FB-Pier (Hoit *et al*., 1996) may be included in this category.

These deficiencies may be removed by means of soil continuum based solutions which are generally based on the finite element (FEM), finite difference (FDM), or boundary element (BEM) methods. These solutions provide an efficient means of retaining the essential aspects of pile interaction through the soil continuum and hence a more realistic representation of the problem. Further, the soil parameters to be introduced into the model have now a clear physical meaning and they can be measured directly.

Programs based on the finite element/finite difference methods

These methods are valuable for clarifying the mechanism of load transfer from the pile to the surrounding soil but, particularly for 3D problems such as pile groups, are not readily applicable to routine design. Apart from the complexity (for example, in relation to the modelling of the pile-soil interfaces) and the considerable effort of data preparation, the main problem is the high computational cost required by this type of analyses, particularly if non-linear soil behaviour is to be considered. This precludes the routine use of such techniques in design (a 3D non-linear analysis of a pile group using current software packages can take several days, even on modern computers running at 800 MHz).

Programs based on the boundary element method

A practical compromise between the approximations of load-transfer approaches and the disproportionate complexity of FEM/FDM solutions is provided by the boundary element method, in which the characteristics of the soil response are represented in a lumped form by ascribing the behavioural features of the soil to the pile-soil interface elements. While the FEM and the FDM require a very large number of elements to model the piles and the surrounding soil by means of 3D meshes, the BEM only requires discretisation of the pile-soil interface, with enormous savings in computational time and data preparation effort.

In the boundary element method, remarkably few elements are required to achieve accuracy of results. A typical BEM mesh for a single pile is shown in Fig. 1, involving discretization of the pile-soil interface into a number of cylindrical elements. The behaviour of each element is considered at one node which is located at the mid-height of the element. This, in practice, reduces the dimensionality of the problem by one and makes 3D modelling a realistic proposition, even for large pile groups.

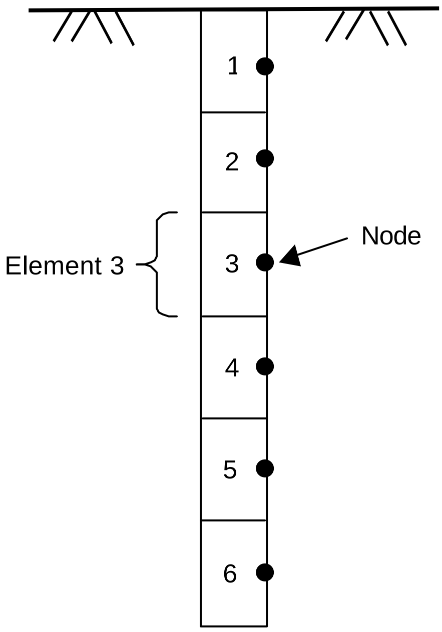

Fig. 1. Typical BEM mesh for single pile

The computer programs DEFPIG (Poulos, 1990) and MPILE, originally developed by Randolph (1980) under the name of Piglet, may be included in this category. These programs are based on simplified BEM analyses which solve the group problem by calculating the influence coefficients for each pair of piles and by merely superimposing the effects. However, it has long been recognised that this approximate procedure produces a number of limitations, in particular it ignores the stiffening effect of intervening piles in a group, thereby leading to an overestimation of interaction between piles.

The above limitation on the use of interaction factors may be removed by simultaneous consideration of all the piles within the group, i.e. performing a "complete" analysis of the group. The computer program PGROUP, originally developed by Banerjee & Driscoll (1976), is included in this category but is restricted to linear elastic analyses and problems of small dimensions because of very large computational resources required. The latter aspect makes the program inapplicable in routine design.

PGroupN analysis method

Repute's calculation engine is the program PGroupN (Basile, 1999, 2003). Its main feature lies in its capability to provide a complete 3D non-linear BEM solution of the soil continuum while retaining a computationally efficient code.

Following the typical BEM scheme, the PGroupN analysis adopts a substructuring technique in which the piles and the surrounding soil are modelled separately and then compatibility and equilibrium conditions at the pile-soil interface are imposed. The soil is modelled using the well-established solution of Mindlin (1936), while the piles are modelled using the classical Bernoulli-Euler beam theory. Thus, given unit boundary conditions, the pile and soil equations are combined together and solved, thereby leading to the distribution of stresses, loads and moments in the piles for any loading condition.

The external group loads are applied incrementally and, at each increment, a check is made that the stress state at the pile-soil interface does not violate the yield criteria. This is achieved by specifying the limiting stresses for the soil according to the classical equations for the axial and lateral pile shaft capacity, and end-bearing resistance. The elements of the pile-soil interface which have yielded can take no additional load and any increase in load is therefore redistributed between the remaining elements until all elements have failed. Thus, by successive application of loading increments, the entire load-displacement relationship for the pile group is determined. Further details on the theoretical formulation of PGroupN are given in Basile (2003).

Choice of soil parameters

The choice of soil parameters for PGroupN is simple and direct: for a linear analysis, it is only necessary to define two soil parameters whose physical interpretation is clear, i.e. the soil modulus (E_s) and the Poisson's ratio (n_s). If the effects of soil non-linearity are considered, the strength properties of the soil need also to be specified, i.e. the undrained shear strength (C_u) for cohesive soils and the angle of friction (f') for cohesionless soils. Thus, the proposed method, by taking into account the continuous nature of pile-soil interaction, removes the uncertainty of empirical *t-z* and *p-y* approaches and provides a simple design tool based on conventional soil parameters.

Non-linear soil model

The PGroupN analysis adopts a non-linear soil model, which follows the well-established hyperbolic relationship between soil stress and strain proposed by Duncan & Chang (1970) and also applied to pile problems by Poulos (1989) and Randolph (1994). This simple relationship assumes that the soil Young's modulus (E_{tan}) varies with the stress level at the pile-soil interface, i.e. it is a function of the initial tangent soil modulus (E_i), the hyperbolic curve-fitting constant (R_f), the current pile-soil stress (t) and the limiting value of pile-soil stress (t_{lim}), as shown in Figure 2. The hyperbolic curve fitting constant R_f defines the degree of non-linearity of the stress-strain response and can range between zero (an elastic-perfectly plastic response) and one (an asymptotic hyperbolic response in which the limiting pile-soil stress is never reached). The best way to determine the value of R_f is by fitting the PGroupN load-deformation curve with the data from the full-scale pile load test. In the absence of any test data, the value of R_f can be initially estimated based on experience.

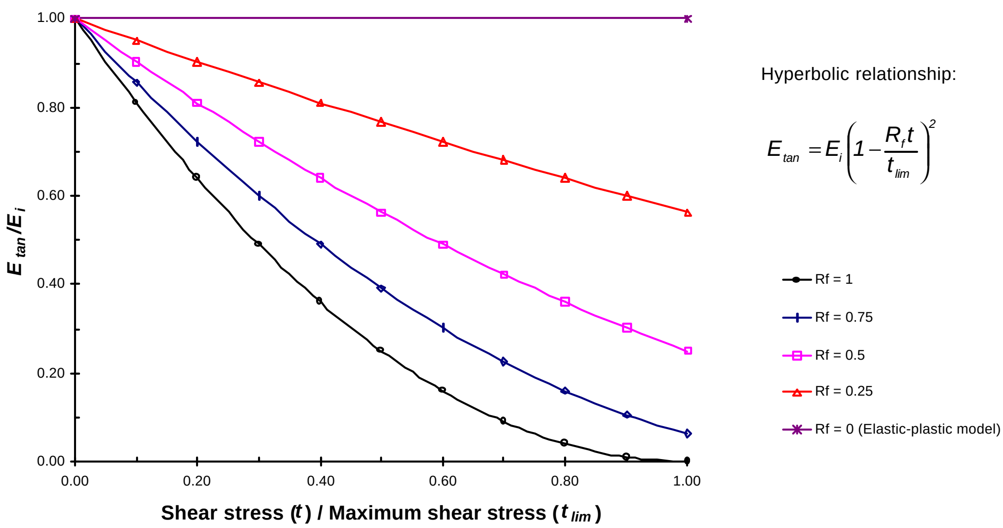

Fig. 2. Soil Young's modulus variation with stress level

Figure 3 shows a typical example of how the non-linear soil model of PGroupN leads to a more realistic predictions of pile response and a better fit with the observed behaviour than traditional linear elastic or elastic-plastic models.

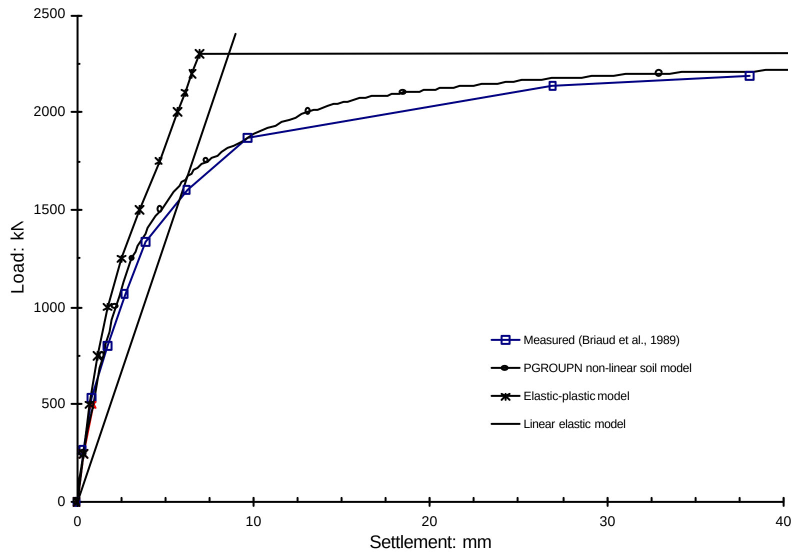

Fig. 3. Load-settlement behaviour of 5-pile group in sand

Another fundamental limitation of the linear elastic models is that they result in a considerable overestimation of the load concentration at the outer piles of the group, and this may lead to an overconservative design. Indeed, it has long been recognised that consideration of soil non-linearity results in a relative reduction of the load concentration at the corner piles and a more uniform load distribution between the piles. It has been shown that, even at typical working load levels, this reduction is significant. This aspect is therefore of basic importance in pile group design (which is strongly influenced by the high corner loads and moments predicted by linear elastic models) and offers the prospect of more effective design techniques and significant savings in construction costs.

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