Chapter 10

Analysis and design of pile groups

F. Basile

Introduction

In his 2000 Rankine Lecture, Professor Atkinson has emphasised the importance of considering soil non-linearity in routine design. For pile group problems, this issue has not yet been satisfactorily addressed, and current design practice is still generally based on linear approaches. The main drawback to the application of linear models to pile group problems is that they ignore the non-linear load-deformation characteristics of soil and hence misrepresent the forces in piles, specifically by giving higher stresses in group corners. The cost of this in practice is high and there is an urgent need in industry for efficient non-linear analysis methods.

An attempt at removing these limitations is represented by the load-transfer approach which is the most widely adopted technique for the non-linear analysis of single piles. However, this approach suffers from some significant restrictions when extended to pile group problems.

A more practical non-linear approach for the analysis of pile groups under general loading conditions (i.e. vertical loads, horizontal loads and moments) has recently been proposed by Basile (1999) and some further developments and applications of the method are described in this chapter. A review of available computer programs for pile group analysis is presented, including some applications in both the linear and non-linear range. The critical question of estimation of geotechnical parameters is addressed, and attention is focused on correlations between these parameters and commonly available *in situ* test data. Finally, attention is turned to the application of available numerical methods to practical problems involving real soils. A number of published case histories are considered, and the predictions from selected methods of analysis are compared with the field measurements.

Numerical methods for pile group analysis

Estimation of the deformations and load distributions in a group of piles subjected to general loading conditions normally requires the use of computer-based methods of analysis. Numerical techniques for pile group analysis may be broadly classified into the following two categories:

(a) continuum-based approaches;
(b) load-transfer (or subgrade reaction) approaches.

The latter category, based on Winkler spring idealisation of the soil, employs load-transfer functions to represent the relationship between the load at any point along the pile and the associated soil deformation at that point. Such a semi-empirical method is widely adopted for the analysis and design of single piles, especially where non-linear soil behaviour has to be considered and/or soil stratification is complicated (e.g. the "*t-z*" or "*p-y*" curve methods of analysis). The computer programs PILGP1 (O'Neill *et al*., 1977), FLPIER (Hoit *et al*., 1996) and GROUP (Reese *et al*., 2000) are included in this category. The main limitations associated with this approach are as follows:

1 The modulus of subgrade reaction is not an intrinsic soil property but instead gives the overall effect of the soil continuum as seen by the pile at a specific depth, and hence its value will depend not only on the soil properties but also on the pile properties and loading conditions. Thus, no direct tests can be conducted to establish force–displacement relationships for that particular pile and soil type, and hence these curves have to be derived from the data obtained by conducting a field test on an instrumented pile. However, due to the high costs, such a test is rarely justifiable for onshore applications and hence standard load-transfer curves are usually adopted in practice. This implies that a significant amount of engineering judgement is needed when formulating these curves for site conditions which differ markedly from the recorded field tests. Murchison and O'Neill (1984) have compared four commonly adopted procedures for selecting *p-y* curves with data from field tests, and their results show that errors in pile-head deflection predictions could be as large as 75%. Huang *et al*. (2001) employed several sets of *p-y* curves derived from DMT data for the analysis of laterally loaded piles, and none of the *p-y* curves yielded reasonable predictions of the measured pile deflections.
2 The load-deformation relationship along the pile is modelled using discrete independent springs and no information is available from the analysis regarding the deformation pattern around the pile. Disregarding continuity through the soil makes it impossible to find a rational way to quantify the interaction effects between piles in a group. Thus, in evaluating group effects, recourse is made to an entirely empirical procedure in which the single pile load-transfer curves are modified on the basis of small-scale and full-scale experiments performed on pile groups in different types of soil. Although Reese and Van Impe (2001) reported some successful analyses of this kind for pile groups under lateral loading, the uncertainties on the general use of the approach in routine design remain (Rollins *et al*., 1998; Rollins *et al*., 2000; Huang *et al*., 2001).
3 It is uncertain how the *p-y* curves are influenced by pile-head fixity. To date, this issue has hardly been addressed, although Reese *et al*. (1975) showed

that the *p-y* relationships are affected by pile-head fixity. The relevance of this aspect is obvious if the *p-y* curves from single pile tests are to be used for pile group predictions where the pile-heads are restrained by a cap.

In conclusion, the load-transfer approach may be regarded as a link between the interpretation of full-scale pile tests and the design of similar piles rather than a general design tool for pile group predictions.

Several hybrid approaches which combine a load-transfer analysis for single pile response and a continuum model to estimate pile–soil–pile interaction have been proposed (Chow, 1986a, 1987; Mandolini and Viggiani, 1997). However, such analyses do not overcome the main limitation of the load-transfer approach that is the questionable assessment of the empirical constants which define the non-linear relationship on the basis of intrinsic soil properties.

The above shortcomings may be removed by means of soil continuum based solutions which are generally based on the finite element method (FEM) (Ottaviani, 1975) or the boundary element method (BEM) (Butterfield and Banerjee, 1971). These solutions provide an efficient means of retaining the essential aspects of pile interaction through the soil continuum and hence a more realistic representation of the problem. Further, the mechanical characteristics to be introduced into the model now have a clear physical meaning and they can be measured directly. Finite element analyses are valuable for clarifying the mechanism of load transfer from the pile to the surrounding soil but, especially for pile groups, are not readily applicable to practical problems. The considerable effort of data preparation and the high computational cost (particularly if non-linear soil behaviour is to be considered) preclude the routine use of such techniques in design. Some idea of the computational resources required may be obtained from the non-linear FEM analysis of a laterally loaded 9-pile group by Kimura and Adachi (1996) who reported a CPU time of 85 hours on a SPARC II work-station.

By contrast, BEM provides a complete problem solution in terms of boundary values only, specifically at the pile–soil interface. This leads to a drastic reduction in unknowns to be solved for, thereby resulting in substantial savings in computing time and data preparation effort. This feature is particularly important for three-dimensional problems such as pile groups.

The following computer programs may be included in this category. DEFPIG (Poulos, 1990), based on a simplified BEM analysis and the use of interaction factors, models soil non-linearity in an approximate manner by means of an elastic-plastic interface model. Two main shortcomings are associated with this model: (1) the non-linear features of stress–strain behaviour are not captured until the load corresponding to the yield of the first interface element is reached; (2) deformations are often seriously underestimated at high load levels. An alternative approach is offered by the widely used computer program MPILE, originally developed by Randolph (1980) under the name of PIGLET. The analysis is based on a semi-empirical method which makes use of approximate analytical solutions for single pile response and for interaction between two piles, in which linear elastic soil behaviour is assumed.

It is important to note that the interaction factor approach (such as is employed in DEFPIG and MPILE) solves the group problem by calculating the influence coefficients for each pair of piles and by merely superimposing the effects. This approximate procedure produces a number of limitations: (a) it ignores the stiffening effect of intervening piles in a group, thereby leading to an overestimation of interaction between piles; (b) its use becomes questionable for cases in which not all the piles are identical; (c) it only gives the loads and bending moments at the pile heads, but not their distributions along the piles; these may only be approximated utilising the single pile solutions with the corresponding pile head loads and bending moments.

The above limitations on the use of interaction factors may be removed by simultaneous consideration of all the piles within the group, i.e. performing a "complete" analysis of the group. The computer program PGROUP, originally developed by Banerjee and Driscoll (1976), is included in this category but is restricted to linear elastic analyses and problems of small dimensions because of the very large computational resources required. The latter aspect makes the program inapplicable in normal design. An even more rigorous linear analysis is performed by the numerical code GEPAN (Xu and Poulos, 2000) in which the boundary elements are meshed in partly cylindrical or annular surfaces. The program provides a benchmark for assessing the accuracy of simplified procedures in the linear range and can also analyse loadings induced by ground movements. However, the relatively high computational cost makes questionable its potential use for routine design problems.

The main feature of the proposed PGROUPN program (Basile, 1999) lies in its capability to provide a complete non-linear BEM solution of the soil continuum while retaining a computationally efficient code. One of the main advantages of a non-linear analysis system over a linear one is that it has the desirable effect of demonstrating a relative reduction of the corner loads in pile groups in both the vertical and horizontal senses. This observation is of basic importance in practice, and offers the prospect of significant improvements in design techniques and potential saving of materials. The choice of soil parameters for PGROUPN is simple and direct: for a linear analysis, it is only necessary to define two soil parameters whose physical interpretation is clear, i.e. the soil modulus (E_s) and the Poisson's ratio (ν_s). If the effects of soil non-linearity are considered, the strength properties of the soil also need to be specified, i.e. the undrained shear strength (C_u) for cohesive soils and the angle of friction (ϕ') for cohesionless soils. These parameters are routinely measured in soils investigation. This aspect represents a significant advantage over the *t-z* and *p-y* curve approaches which are based on empirical parameters which may only be derived from the results of pile load tests. However, in many practical situations it is not possible to carry out such testing, at least in the preliminary stages of design.

A summary of the main capabilities and limitations of some of the computer programs discussed above is presented in Table 10.1.

Table 10.1 Capabilities and limitations of various computer programs for pile group analysis

Program name	*PGROUPN*	*MPILE*
Latest version	1.13	1.50
Year	2001	2000
DOS User Interface	Text Interface	Text Interface
Windows User Interface	Graphical Interface(1)	NA
Max no. of piles	200	100
Max no. of pile elements	50	No pile discretization
Loading	Vertical, Horizontal and Moment (Note: Horizontal loads and Moments acting in two directions)	Vertical, Horizontal, Moment and Torsional (Note: Horizontal loads and Moments acting in two directions)
General output	Cap displacements and rotations; profiles of pile shear/normal stresses, axial/lateral loads and moments	Cap displacements and rotations; axial/lateral loads and moments at pile heads only; approximate profiles of moments
Analysis method	Complete BEM solution	Semi-empirical analysis using interaction factors
Soil model	Linear or Non-linear (using hyperbolic continuum-based interface model)	Linear
Soil profile	Multi-Layered	Homogeneous or Gibson
Soil layer	Finite or Semi-infinite	Semi-infinite
Soil modulus	Independent profiles for axial and lateral loading	Independent profiles for axial and lateral loading
Cap stiffness	Fully rigid	Fully rigid or Fully flexible (for vertical loading only)
Cap-soil contact	Effective(1) or Non-effective	Non-effective
Pile modulus	Can vary	Same for all
Pile lengths	Can vary	Same for all
Pile shaft diameters	Can vary	Can vary
Pile base diameters	Can vary	Can vary
Pile rake	In two directions	In two directions
Pile-head fixity at cap	Rigidly fixed	Rigidly fixed or Pinned

Notes: NA = Not applicable; (1) = Under development; (2) = Finite difference discretisation.

PGROUP	*DEFPIG*	*GROUP*
3.0	1.6	5.0
1981	1990	2000
NA	NA	NA
NA	NA	Graphical Interface
200	36	100
11	26 (under vertical loading), 50 (under horizontal loading)	100(2)
Vertical, Horizontal and Moment in one direction	Vertical, Horizontal and Moment in one direction	Vertical, Horizontal, Moment and Torsional (Note: Horizontal loads and Moments acting in two directions)
Cap displacements and rotation; normal stresses at cap-soil interface; profiles of pile shear/normal stresses, axial/lateral loads and moments	Cap displacements and rotation; approximate profiles of pile , displacements shear/normal stresses, axial/lateral loads and moments	Cap displacements and rotations; profiles of pile displacements, shear/normal stresses, axial/lateral loads and moments
Complete BEM solution	Simplified BEM analysis using interaction factors	Load-transfer approach (Winkler spring model)
Linear	Linear or Non-linear (approximated using elastic-plastic interface model)	Non-linear (using *t-z*, *q-w* and *p-y* curves)
Homogeneous, Gibson or Two-Layered	Multi-Layered	Multi-Layered
Semi-infinite	Finite or Semi-infinite	NA
Same profile for axial and for lateral loading	Independent profiles for axial and lateral loading	NA
Fully rigid	Fully rigid or Fully flexible	Fully rigid
Effective or Non-effective	Effective or Non-effective	Non-effective
Same for all	Same for all	Can vary
Can vary	Same for all	Can vary
Can vary	Same for all	Can vary
Can vary	Same for all	Equal to shaft diameters
In one direction	In one direction	In two directions
Rigidly fixed	Rigidly fixed or Pinned	Rigidly fixed, Pinned or Restrained

Load distribution in pile groups

The distribution of load between piles in a group is of basic importance in design. When a group of piles connected by a rigid "free-standing" cap (a common assumption for this kind of problem) is subjected to a system of vertical loads, horizontal loads and moments, the following features of behaviour play a major role in the prediction of the load distribution between the piles:

1 *Pile-to-pile interaction*
Due to pile-to-pile interaction, groups of piles tend to deform more than a proportionally loaded single pile. This is because neighbouring piles are within each others' displacement fields and hence the load per pile to generate a given displacement is reduced for the central piles and increased for the outer ones. Therefore, in a group of piles, the distribution of load is not uniform, i.e. the corner piles carry the greatest proportion of load, while those near the centre carry least. This feature of behaviour is commonly modelled using the interaction factor approach (e.g. in MPILE and DEFPIG). However, as discussed previously, this approximate method suffers from some significant limitations.

2 *Group stiffening effect*
The simultaneous presence of all the piles within the soil mass has the effect of "stiffening" the soil continuum. Therefore, the central pile of a group (the most affected by the presence of the other piles) is subjected to a reduction of the head deformation due to the greater stiffness of the surrounding soil, "reinforced" by the presence of the other piles. This increased stiffness of the central pile results in a higher proportion of the applied load taken by the pile and hence the non-uniformity of load distribution resulting from pile-to-pile interaction (Feature No. 1) is reduced. It has been shown that these group stiffening effects are more marked in a laterally loaded pile group than in an axially loaded one (Burghignoli and Desideri, 1995; Basile, 1999), and they become more significant for increasing the number of piles in a group.

It is therefore important to recognise that each pile interacts with the surrounding soil with a twofold effect: on the one hand, the displacement of the other piles tends to increase as a result of the stresses transferred to the surrounding soil (Feature No. 1); this increase may be expressed in terms of "interaction factors". On the other hand, by reinforcing the continuum in which the piles are located, the effects of interaction with the other piles are decreased (Feature No. 2). The latter aspect cannot be reproduced in the interaction factor method and it can only be accounted for by a "complete" approach.

3 *Load-deformation coupling*
Pile–soil interaction is a three-dimensional problem, and each of the load components has deformation-coupling effects, i.e. there is an interaction between the axial and lateral response of the piles. Modelling of this aspect becomes important when a pile group is subjected to a combination of vertical and horizontal loads. In this case, only a proper consideration of the interaction between the axial and lateral response will lead to a realistic estimate of the

loads acting on the piles, which will be increased for the piles in the leading rows and decreased for those in the trailing rows of the group. However, in current design practice, such interaction effects are not properly accounted for, and the axial and lateral responses of the piles are treated separately.

4 *Soil non-linearity*
A fundamental limitation of the linear elastic methods is that they result in a considerable overestimation of the load concentration at the outer piles of the group, and this may lead to an overconservative design. Indeed, it has long been recognised that consideration of soil non-linearity results in a reduction of the stiffness of the piles, the reduction being greater for piles at a greater load level, i.e. for the corner piles. Consequently, as the total applied load increases, the share of the load carried by the corner piles progressively decreases. This results in a redistribution of the loads in the individual piles, leading to a more uniform distribution than that predicted by linear models. Ideally, for an axially loaded pile group, all piles will carry the same load as the total applied load approaches the ultimate load capacity of the group.

Table 10.2 summarises the above-mentioned features and their effect on the prediction of load at group corners. The table also shows the ability of the computer programs discussed above to model such aspects of group behaviour. It is worth noting that all the features mentioned above may be modelled using the PGROUPN analysis, whereas the other programs can only model some of these aspects, thereby neglecting important features of group behaviour. There is thus a number of compelling arguments for adopting a design methodology which deals with group effects on a more fundamental basis.

PGROUPN method of analysis

The PGROUPN analysis is based on a complete non-linear BEM formulation, extending an idea first proposed by Butterfield and Banerjee (1971) and incorporated into a number of computer programs, including PGROUP (Banerjee and Driscoll, 1976), GAPFIX (Poulos and Hewitt, 1986) and that developed by

Table 10.2 Features of group behaviour and their effect on corner loads

Features of group behaviour	*Effect on corner load*	*PGROUPN*	*MPILE*	*PGROUP*	*DEFPIG*	*GROUP*
(1) Pile-to-pile interaction	↑	x	x	x	x	x
(2) Group stiffening effect	↓	x		x		
(3) Loading-deformation coupling	↑	x		x		
(4) Soil non-linearity	↓	x			x(1)	x

Note
x indicates capability; (1) = Using elastic-plastic soil model

Burghignoli and Desideri (1995). The analysis involves discretisation of only the pile–soil interface into a number of cylindrical elements, while the base is represented by a circular (disc) element. The method employs a substructuring technique in which the piles and the surrounding soil are considered separately and then compatibility and equilibrium conditions are imposed at the interface. A description of the basic theoretical formulation of the PGROUPN analysis has been presented elsewhere (Basile, 1999) and hence only a brief description will be given here. However, the additional features that have recently been introduced will be described in some detail.

Soil domain

The boundary element method involves the integration of an appropriate elementary singular solution for the soil medium over the surface of the problem domain, i.e. the pile–soil interface. With reference to the present problem, the well-established solution of Mindlin (1936) for a point load within a homogeneous, isotropic elastic half-space has been adopted, yielding:

$$\{u_s\} = [G_s]\{t_s\} \tag{10.1}$$

where $\{u_s\}$ are the soil displacements, $\{t_s\}$are the soil tractions and $[G_s]$ is the flexibility matrix obtained from Mindlin's solution. The singular part of the $[G_s]$ matrix is calculated via analytical integration of the Mindlin functions. This is a significant advance over previous work (e.g. PGROUP) where these have been integrated numerically, since these singular integrals require considerable computational resources.

Treatment of multi-layered soil profiles

Mindlin's solution is strictly applicable to homogeneous soil conditions. However, in practice, this limitation is not strictly adhered to, and the influence of soil non-homogeneity is often approximated using some averaging of the soil moduli. PGROUPN handles multi-layered soils according to the averaging procedure first examined by Poulos (1979) and widely accepted in practice (Chow, 1986a, 1987; Poulos, 1989, 1990; Xu and Poulos, 2000); in the evaluation of the influence of one loaded element on another, the value of the soil modulus is taken as the mean of the values at the two elements. This procedure is adequate in most practical cases but becomes less accurate if large differences in soil modulus exist between adjacent elements or if a soil layer is overlain by a much stiffer layer (Poulos, 1989).

Finite soil layer

Mindlin's solution has been used to obtain approximate solutions for a layer of finite thickness by employing the Steinbrenner approximation (Steinbrenner, 1934) to allow for the effect of the underlying rigid base in reducing the soil displacements (Poulos and Davis, 1980; Poulos, 1989).

Pile domain

If the piles are assumed to act as simple beam-columns which are fixed at their heads to the pile cap, the displacements and tractions over each element can be related to each other via the elementary beam theory, yielding:

$$\{u_p\} = [G_p]\{t_p\} + \{B\} \tag{10.2}$$

where $\{u_p\}$ are the pile displacements, $\{t_p\}$ are the pile tractions, $\{B\}$ are the pile displacements due to unit boundary displacements and rotations of the pile cap, and $[G_p]$ is a matrix of coefficients obtained from the elementary (Bernoulli–Euler) beam theory.

Solution of the system

The soil and pile equations (10.1) and (10.2) may be coupled via compatibility and equilibrium constraints at the pile–soil interface. Thus, by specifying unit boundary conditions, i.e. unit values of vertical displacement, horizontal displacement and rotation of the pile cap, these equations are solved, thereby leading to the distribution of stresses, loads and moments in the piles for any loading condition.

Limiting pile–soil stresses

It is essential to ensure that the stress state at the pile–soil interface does not violate the yield criteria. This can be achieved by specifying the limiting stresses for the soil.

Cohesive soil

For cohesive soils, a total stress approach is adopted. The limiting shear stress in the slip zone (i.e. the pile shaft for the axial response) is taken as:

$$t_{ss} = \alpha C_u \tag{10.3}$$

where C_u is the undrained shear strength of the soil and α is the adhesion factor. The limiting bearing stress on the pile base is calculated as:

$$t_{sc} = 9C_u \tag{10.4}$$

The limiting bearing stress on the pile shaft for the lateral response is calculated as:

$$t_{sc} = N_c\, C_u \tag{10.5}$$

where N_c is a bearing capacity factor increasing linearly from 2 at the surface to a constant value of 9 at a depth of three pile diameters and below, much as was originally suggested by Broms (1964) and widely accepted in practice (Fleming *et al*., 1992).

Cohesionless soil

For cohesionless soils, an effective stress approach is adopted. The limiting shear stress in the slip zone (i.e. the pile shaft for the axial response) is taken as:

$$t_{ss} = K_s \sigma'_v \tan\delta \tag{10.6}$$

where K_s is the coefficient of horizontal soil stress, σ'_v is the effective vertical stress and δ is the angle of friction between pile and soil. The limiting bearing stress on the pile base is calculated as:

$$t_{sc} = N_q \sigma'_v \tag{10.7}$$

where N_q is calculated as a function of the soil angle of friction (ϕ') and the length-to-diameter ratio (*L/d*) of the pile, much as was originally established by Berezantzev *et al*. (1961). The limiting bearing stress on the pile shaft for the lateral response is calculated as (Fleming *et al*., 1992):

$$t_{sc} = K_p^2 \sigma'_v \tag{10.8}$$

where K_p is the passive earth pressure coefficient, equal to $(1+\sin\phi')/(1-\sin\phi')$.

Group "shadowing" effect

Under lateral loads, closely spaced pile groups are subjected to a reduction of lateral capacity. This effect, commonly referred to as "shadowing", is related to the influence of the leading row of piles on the yield zones developed in the soil ahead of the trailing row of piles. Because of this overlapping of failure zones, the front row will be pushing into virgin soil while the trailing row will be pushing into soil which is in the shadow of the front row piles. A consequence of this loss of soil resistance for piles in a trailing row is that the leading piles in a group will carry a higher proportion of the overall applied load than the trailing piles. This effect also results in gap formation behind the closely spaced piles and an increase in group deflection. It has been shown both theoretically and experimentally that the shadowing effect becomes less significant as the spacing between piles increases and is relatively unimportant for centre-to-centre spacing greater than about six pile diameters (Cox *et al*., 1984; Brown and Shie, 1990; Ng *et al*., 2001).

The shadowing effect has been modelled into the PGROUPN analysis using the approach outlined by Fleming *et al*. (1992). Following this approach, it has been assumed that a form of block failure will govern when the shearing resistance of the soil between the piles is less than the limiting resistance of an isolated pile. Referring to Figure 10.1, the limiting lateral resistance for the pile which is in the shadow of the front pile may be calculated from the lesser of the limiting bearing stress for a single pile (as calculated from Equations (10.5) and (10.8)) and $2\frac{s}{d}t_s$, where s is the centre-to-centre pile spacing, d is the pile diameter and t_s is the friction on the sides of the block of soil between the two piles. The value of t_s may be taken as C_u for cohesive soil and $\sigma'_v \tan\phi'$ for cohesionless soil.

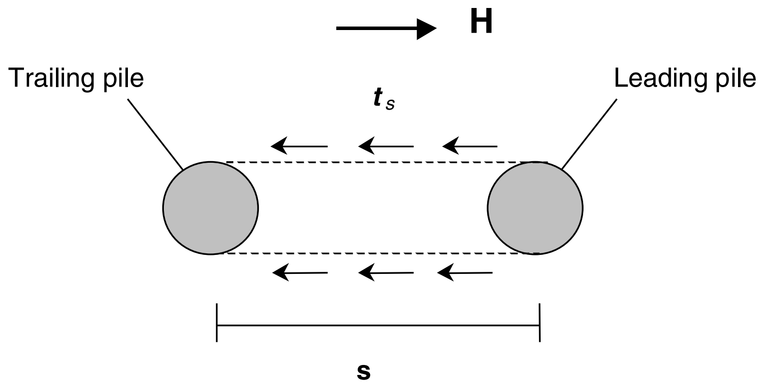


Figure 10.1 Plan view of block failure under lateral load (after Fleming et al., 1992).

The outlined approach provides a simple yet rational means of estimating the shadowing effect in closely spaced groups, as compared with the purely empirical "*p*-multiplier" concept which is employed in load-transfer analyses (e.g. in GROUP and FLPIER).

Extension to non-linear soil behaviour

Non-linear soil behaviour is incorporated, in an approximate manner, by assuming that the soil Young's modulus varies with the stress level at the pile–soil interface. A simple and popular assumption is to adopt a hyperbolic stress–strain relationship, in which case the tangent Young's modulus of the soil E_{tan} may be written as (Duncan and Chang, 1970; Poulos, 1989; Randolph, 1994):

$$E_{tan} = E_i \left(1 - \frac{R_f t}{t_{lim}}\right)^2 \tag{10.9}$$

where E_i is the initial tangent soil modulus, R_f is the hyperbolic curve-fitting constant, t is the pile–soil stress and t_{lim} is the limiting value of pile–soil stress obtained from Equations (10.3)–(10.8). Thus, the boundary element equations described above for the linear response are solved incrementally using the modified values of soil Young's modulus of Equation (10.9) and enforcing the conditions of yield, equilibrium and compatibility at the pile–soil interface.

The hyperbolic curve fitting constant R_f defines the degree of non-linearity of the stress–strain response and can range between 0 (an elastic–perfectly plastic response) and 1.0 (an asymptotic hyperbolic response in which the limiting pile–soil stress is never reached). Different values of R_f should be used for the axial response of the shaft and the base, and for the lateral response of the shaft. For the axial response of the shaft, there is a relatively small amount of non-linearity, and values of R_f in the range 0–0.5 are appropriate (Poulos, 1989, 1994; Hirayama, 1991), the higher values being associated with relatively rigid piles. The (axial) response of the base is highly non-linear, and a value of R_f in the range 0.9–1 is recommended (Poulos, 1989, 1994). For the lateral response of the shaft, values of R_f in the range 0.5–1 generally give a reasonable fit with the observed behaviour, the higher values being recommended to avoid underestimation of deflections at high load levels.

Numerical results

The results obtained from alternative numerical methods for single piles and pile groups subjected to vertical and horizontal loads are compared and discussed. Benchmark solutions in the linear and non-linear range are presented, and the significant influence of soil non-linearity on load distribution between individual piles in a group is highlighted.

Single pile response

In comparing non-linear solutions for single pile response to axial loading, the problem examined is that reported by Poulos (1989) in his Rankine Lecture. This example offers the opportunity to assess the validity of the non-linear hyperbolic model adopted by PGROUPN by comparison with well-established numerical solutions. The input parameters are reported in Table 10.3 and, in order to cover a wide range of pile–soil relative stiffnesses ($K = E_p / E_s$), two values of pile Young's modulus have been considered, 30GPa and 30,000GPa (the latter would be unrealistically stiff in practice). Figures 10.2 and 10.3 report the pile head load–settlement response obtained from a FEM analysis by Jardine *et al*. (1986) which can be used as a benchmark. Such analysis involves the use of a non-linear soil model in which the Young's modulus decreases markedly from an initial value of 1056 MPa as the axial strain level increases. Figures 10.2 and 10.3 also show the load–settlement curves obtained from the following two BEM analyses by Poulos (1989): (a) an elastic–perfectly plastic continuum-based interface model, using a constant soil Young's modulus of 1056 MPa; (b) a hyperbolic non-linear continuum-

Table 10.3 Parameters for the analyses reported in Figures 2–4

Parameter	*Value*
Pile length, *L* (m)	30
Pile diameter, *d* (m)	0.75
Depth of soil layer (m)	50
Pile Young's modulus, E_p (GPa)	30, 30000
Soil Young's modulus, E_s (MPa)	1056
Soil Poisson's ratio, ν_s	0.49
Limiting shear stress, t_{ss} (kPa)	220

based interface model (similar to PGROUPN), using an initial tangent soil Young's modulus of 1056 MPa and a hyperbolic curve fitting constant (R_f) of 0.9 for both the shaft and the base. The PGROUPN solutions have been obtained for three sets of hyperbolic curve fitting constants: (1) $R_f = 0.5$ for the shaft and $R_f = 0.9$ for the base (this set attempts to reproduce the FEM results); (2) $R_f = 0$ for both the shaft and the base (to be compared with curve (a) by Poulos); (3) $R_f = 0.9$ for both the shaft and the base (to be compared with curve (b) by Poulos).

It is worth noting that, for the more compressible and realistic pile (Figure 10.2), all BEM analyses (perhaps excluding the analyses including $R_f = 0.9$ for both the shaft and the base) are capable of predicting a very similar load–settlement response to that obtained from the FEM solution which utilises a non-linear constitutive model of soil behaviour. For the stiffer pile (Figure 10.3), the agreement between the curves is not as close, and only the PGROUPN analysis using $R_f = 0.5$ for the shaft and $R_f = 0.9$ for the base is in good agreement with the FEM solution. It is clear that, for very stiff piles, the details of the pile–soil interface model have a greater influence on the load–settlement response than for more compressible piles. For this type of problem, two features of behaviour are worthy of note: (1) the elastic-perfectly plastic model, such as is employed in curve (a) (and also in DEFPIG), is not capable of capturing the non-linear features of stress–strain behaviour; (2) the use of $R_f = 0.9$ for the shaft within a hyperbolic non-linear model leads to a significant overprediction of pile settlements, especially at high load levels.

Finally, Figure 10.4 reports the mobilisation of shaft resistance t_s / C_u for a factor of safety (FoS) of 2 (i.e. at a load level $P / P_u = 0.5$, where P is the applied

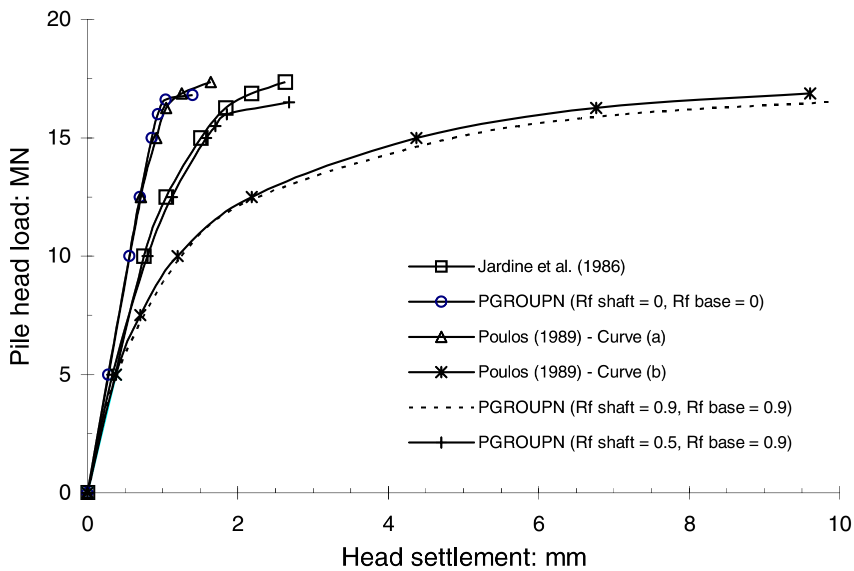


Figure 10.2 Comparison of load–settlement response for single pile (E_p = 30 GPa).

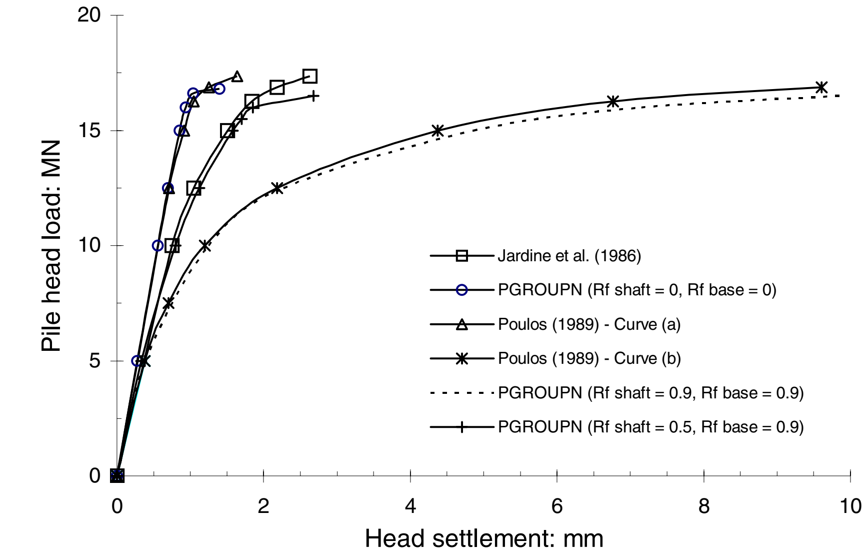


Figure 10.3 Comparison of load-settlement response for single pile (E_p = 30,000 GPa).

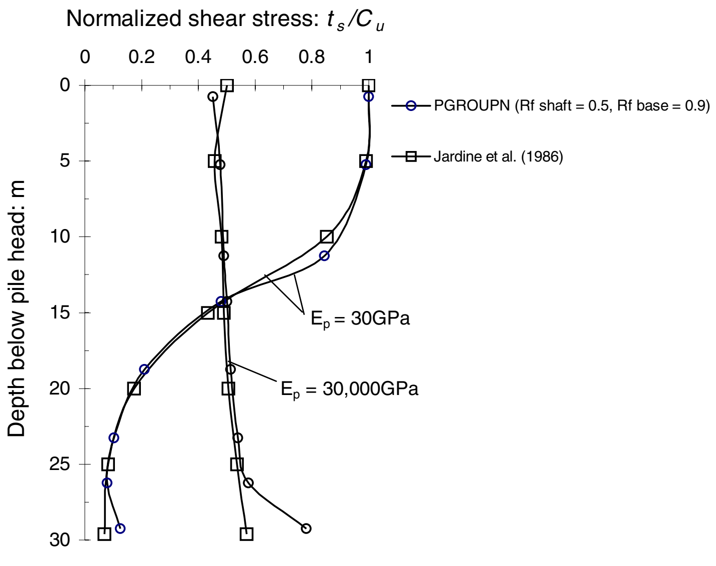


Figure 10.4 Comparison of shaft shear stress distribution for single pile at a load level $P/P_u = 0.5$.

axial load and P_u is the ultimate axial capacity of the pile). The results show that the distribution of shear stress (t_s) predicted by PGROUPN (using $R_f = 0.5$ for the shaft and $R_f = 0.9$ for the base) is very consistent with that obtained from the FEM analysis of Jardine and colleagues.

Pile group settlement

In order to investigate pile group settlement predictions in the linear range, Figure 10.5 compares PGROUPN results with those obtained by some of the computer programs mentioned above. Results are expressed in terms of the normalised group stiffness $k_p/(\sqrt{n}sG)$ of square groups of piles at different spacings (where k_p is the ratio of the total vertical load acting on the group to the average settlement of the group, n is the number of piles in the group, s is the pile spacing and G is the soil shear modulus). In the analyses of PIGLET and GRUPPALO, it is assumed that axial interaction effects between piles become insignificant for a pile spacing greater than a limiting value s_{max} equal to (Randolph and Wroth, 1979):

$$s_{max} = 2.5L(1-\nu_s) + r_g \tag{10.10}$$

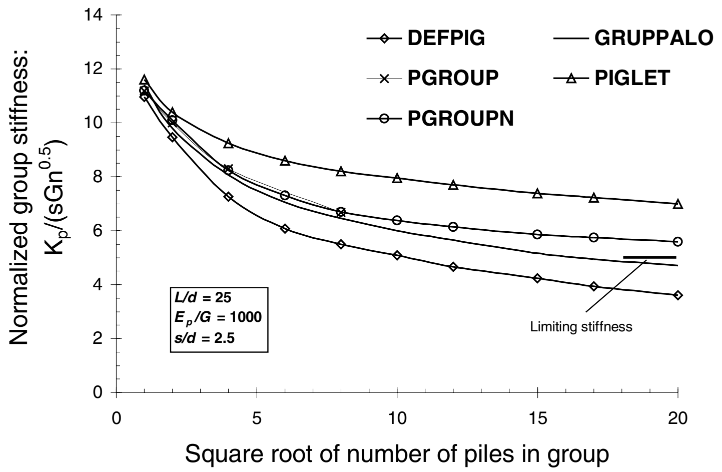
where ν_s is the soil Poisson's ratio and r_g, for rectangular pile group configurations, may be taken as the radius of the circle of equivalent area to that covered by the pile group. In the analyses of DEFPIG, PGROUP and PGROUPN, no limiting value for axial interaction effects has been adopted.

It may be observed that DEFPIG and PIGLET approaches give divergent results, while a reasonable agreement is obtained between PGROUPN and the computer program GRUPPALO (Mandolini and Viggiani, 1997). It is worth noting that results from PGROUPN are in excellent agreement with the rigorous BEM solution of PGROUP, but the latter is limited to groups of 8 × 8 piles, due to the magnitude of computer resources required to analyse larger groups. In contrast, PGROUPN took about 30 CPU s on an ordinary desktop computer for the 20 × 20 pile group, considering the symmetry of the pile arrangement. This observation is of great significance because it demonstrates the applicability of the complete BEM approach to large pile groups, whereas previous work (i.e. PGROUP) was restricted to small pile groups.

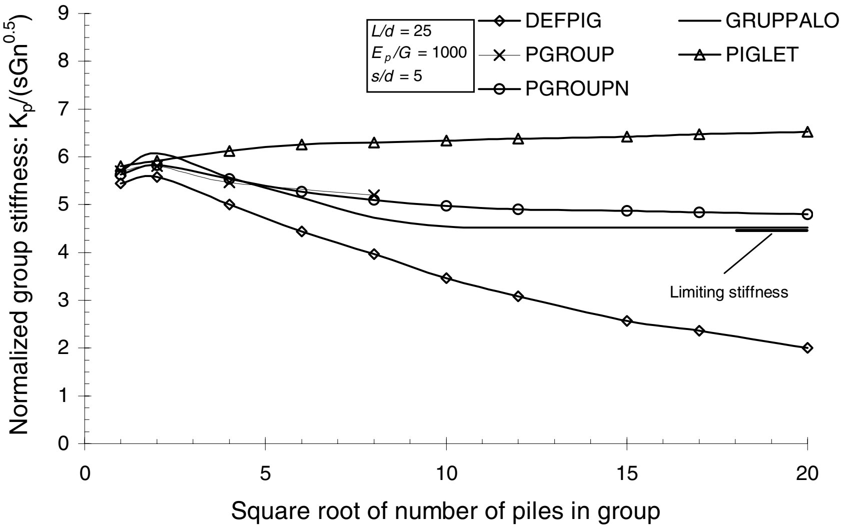
Finally, it may be noted that, for very large pile groups, where the ratio of pile group width to pile length becomes much greater than unity, the group stiffness should approach that of a shallow foundation. This would correspond to a limiting stiffness of about 4.5 (Fraser and Wardle, 1976), as indicated in Figure 10.5.

Axial load distribution

Figure 10.6 shows a comparison of the distribution of axial load in a 5 × 5 pile group in homogeneous soil which has been obtained from selected numerical codes. The results from a simplified BEM analysis using interaction factors by Poulos and Davis (1980) are also included. The load distribution is expressed in terms of



(a)



(b)

Figure 10.5 Comparison of different pile group analysis methods for (a) $s/d = 2.5$; (b) $s/d = 5$.

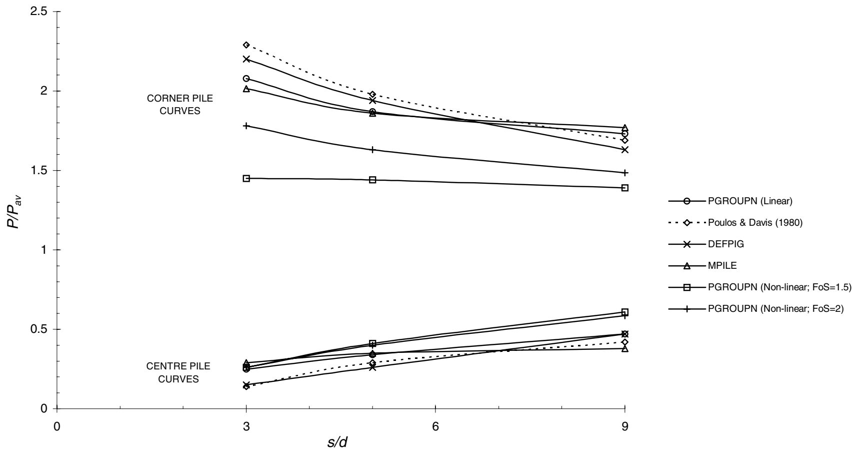


Figure 10.6 Comparison of axial load distribution to individual piles in 5 × 5 pile group.

the ratio of load on pile to the average pile load in the group (P/P_{av}), and is plotted against the normalised pile spacing (*s/d*). The input parameters of the analyses are given in Tables 10.4 and 10.5 for the linear and non-linear range, respectively. In the linear range, the load distribution predicted by PGROUPN compares favourably with that predicted by MPILE, whereas the results of DEFPIG and Poulos and Davis (1980) give slightly higher corner loads.

In the non-linear range, the PGROUPN results have been obtained by applying a total load on the group of 29.0 MN (corresponding to a group FoS of 2.0) and 38.6 MN (corresponding to a group FoS of 1.5). It is evident that consideration of the non-linear soil response yields a significant reduction in the load concentration at the corner piles and a more uniform load distribution. Clearly, the lower the factor of safety, the higher the reduction in the load concentration at the corner piles obtained by the non-linear analysis.

Lateral load distribution

As a pile group is subjected to a lateral load, this will result in a lateral deformation as well as a rotation of the group and hence the piles at the edge will be loaded axially in tension and compression. Thus, only if rotation of the cap is prevented, do the piles deflect purely horizontally and hence the lateral load deformation characteristics of the group can be analysed separately from the axial ones. For such fixed-head pile groups, the lateral load distribution to the individual piles predicted by selected numerical codes is examined by applying a lateral load of 15 MN to the same group of piles analysed in the previous section, and under the same soil conditions (refer to Tables 10.4 and 10.5). Similarly, Figure 10.7 shows the lateral load distribution (where *H* is the load acting on the individual pile

Table 10.4 Parameters for the linear analyses reported in Figures 6–7

Parameter	*Value*
L (m)	25
Pile diameter, d (m)	1
Pile Young's modulus, E_p (GPa)	25
Soil Young's modulus, E_s (MPa)	25
n's ratio, ν_s	0.5

Table 10.5 Additional parameters for the non-linear analyses reported in Figures 6–7

Parameter	*Value*
Initial soil Young's modulus, E_s (MPa)	75
Undrained shear strength, C_u (kPa)	50
Adhesion factor, α	0.5
Hyperbolic curve fitting constant R_f (shaft)	0.5
Hyperbolic curve fitting constant R_f (base)	1.0
Hyperbolic curve fitting constant R_f (lateral)	0.9

head and H_{av} is the average load acting on each pile head) as a function of the normalised pile spacing (s/d).

In the linear range, the PGROUPN solutions compare favourably with DEFPIG, while significant discrepancies with the MPILE analysis are observed in the corner load prediction. These differences may partially be explained with the approximations involved in the interaction factor approach which ignores the stiffening effects of piles within the soil mass, thereby leading to an overestimation of group interaction, as discussed in Section 3. Consideration of soil non-linearity results in a reduction of the load concentration at the corner pile and hence a more uniform load distribution. The amount of this reduction will depend on the load level. Finally, it should be emphasised that the numerical simulations presented herein take no account of possible failure by yielding of the pile section, i.e. the pile is assumed to remain elastic.

Pile group under general loading conditions

The deformations and load distribution in a 3-pile group under a combination of axial load, lateral load and moment are examined in the linear range (refer to Figure 10.8). Results from selected numerical analyses are compared in Table 10.6 in which w_3, u and θ are the vertical head displacement of pile no. 3, the horizontal cap displacement and the rotation of the cap, respectively. There is a good agreement between the solutions which consider pile–soil–pile interaction (even if with different degrees of rigour), whereas the equivalent-bent analysis (reported in Poulos and Davis, 1980) gives quite different results, thereby showing

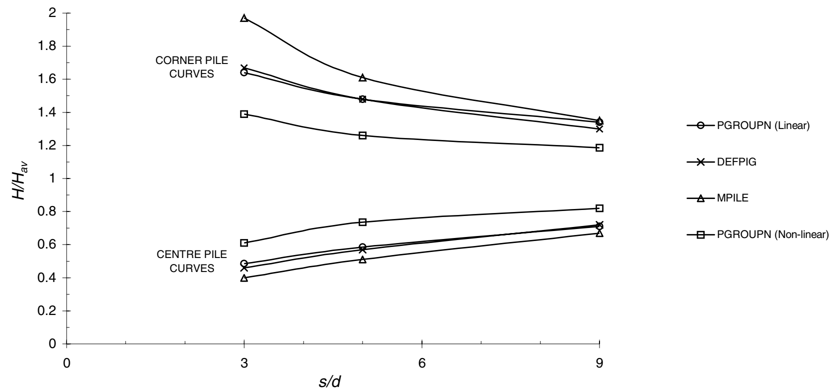


Figure 10.7 Comparison of lateral load distribution to individual piles in 5 × 5 pile group.

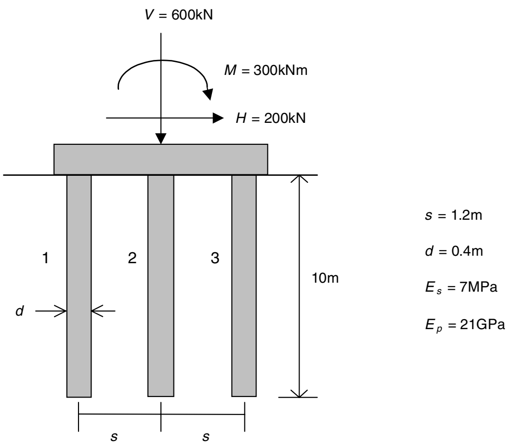


Figure 10.8 Group of 3 piles considered in comparison of methods.

the pitfall of attempting to model a complex pile–soil system by means of a simple structural frame.

Selection of soil parameters

In predicting the behaviour of pile foundations, the designer is faced with a number of decisions, including the selection of the method of analysis and the soil parameters to be adopted. It is crucial to recognise that the latter aspect is

Table 10.6 Comparison of different analyses for 3-pile group under general loading conditions

Quantity	*Equivalent-bent analysis*	*DEFPIG*	*PIGLET*	*GEPAN*	*PGROUPN*
V_1 (kN)	67.2	55.8	55.7	54.0	49.6
V_2 (kN)	200.0	155.1	155.0	156.0	153.0
V_3 (kN)	332.8	389.1	389.3	390.0	397.0
H_1 (kN)	66.8	72.0	80.4	73.7	68.9
H_2 (kN)	66.7	56.0	39.3	50.9	53.5
H_3 (kN)	66.6	72.0	80.4	75.4	77.6
M_1 (kNm)	–6.2	–35.8	–42.0	–38.5	–41.5
M_2 (kNm)	–6.2	–28.5	–16.3	–26.1	–31.8
M_3 (kNm)	–6.2	–35.8	–42.0	–38.6	–44.0
w_3 (mm)	17.5	13.4	9.9	10.8	14.1
u (mm)	8.9	11.6	11.4	10.5	11.5
θ (rad)	0.00581	0.00242	0.00242	0.00241	0.00263

generally of greater importance than the method of analysis, provided that a soundly based method is employed.

Attention will be focused here on the estimation of the soil Young's modulus (E_s), which is the key geotechnical parameter for pile deformation predictions. The most reliable means of determining E_s is by backfiguring from the results of full-scale pile load tests, using the same theory that will be used for the actual deformation prediction. However, this is not always possible, at least in the preliminary stages of design, and hence resort is made to the results of laboratory or *in situ* soil tests.

The PGROUPN analysis is based on a non-linear hyperbolic interface model. For this kind of analysis, previous experience has shown that the initial ("low strain") value of E_s may be successfully employed in the prediction of the initial stiffness of the load–settlement curve of pile foundations (Poulos, 1989; Randolph, 1994; Mandolini and Viggiani, 1997). The use of an initial tangent soil modulus represents an advantage over a purely linear analysis which requires a secant value of soil modulus, relevant for the applied load level. Indeed, selection of an appropriate secant modulus is by no means straightforward, whereas the initial modulus is a more reproducible quantity. Some indication of the typical ratio of secant modulus to initial modulus as a function of the applied load level has been presented by Poulos *et al*. (2001).

It is important to recognise that the value of E_s for the soil in the vicinity of the pile shaft will be influenced by both the loading of the pile and the installation process, and would be expected to be different for bored piles and for driven piles. As discussed by Randolph (1994), for driven piles, the soil modulus may be expected to be higher in the zone immediately around the pile, while for bored piles the soil modulus will be reduced. The near-pile E_s will tend to influence strongly the deformation of the single pile, whereas initial values of E_s will affect interaction effects between piles. Typical values for the near-pile soil modulus for bored and driven piles have been reported by Poulos (1993) and Poulos (1994), respectively. The discussion below will give some indication on the assessment of the initial soil modulus.

It is now well understood that the values of E_s determined from conventional triaxial tests with external measurement of axial strain of the soil sample (which is highly inaccurate at strains less than about 0.1%) are usually much smaller (typically one-fourth to one-tenth) than the initial modulus (Jardine *et al*., 1984). The most reliable means of obtaining the low strain shear modulus (G_o), which is connected with E_s by the formula $E_s = 2G_o(1+\nu_s)$, is to carry out *in situ* shear wave velocity measurements. Mandolini and Viggiani (1997) showed that there is a substantial agreement between low strain shear moduli derived from cross-hole data and those backfigured from pile loading tests, with a trend of the latter to fit the lower limit of the geophysical measurements. If *in situ* shear wave velocity measurements are not available, G_o may be determined in the laboratory using bender elements (Viggiani and Atkinson, 1995).

However, all of these means of measuring shear moduli are expensive and time-consuming, and are rarely available in the early stages of design. Thus, a preliminary assessment of initial soil modulus may be obtained from empirical correlations with the results of conventional *in situ* and laboratory tests.

For clays, a correlation between initial E_s and SPT N-value (blows/300 mm, corrected to a rod energy of 60%) has been proposed by Poulos (1993), based on the work by Wroth *et al*. (1979):

$$E_s = 25N^{0.77} \quad \text{[MPa]} \tag{10.11}$$

For convenience, a linear correlation may also be adopted, as suggested by Hirayama (1991, 1994) and Poulos (1993, 1994):

$$E_s = 14N \quad \text{[MPa]} \tag{10.12}$$

However, it is probably more reliable to correlate the initial soil modulus with the undrained shear strength (C_u), and the following correlation is suggested by Hirayama (1991, 1994) and Poulos (1993, 1994):

$$E_s = 1500C_u \tag{10.13}$$

Several other correlations have been proposed, i.e. E_s = 1500-3000C_u (Jardine *et al*., 1986), E_s = 1200-2700C_u (Kuwabara, 1991), E_s = 1900C_u (Kagawa, 1992). Thus, Equation (10.13) may give average values which are expected to be on the safe side.

For silica sands, the following correlation between initial E_s and SPT N-value may be used (Ohsaki and Iwasaki, 1973; Poulos, 1994):

$$E_s = 16.9N^{0.9} \quad \text{[MPa]} \tag{10.14}$$

Alternatively, the initial soil modulus may be correlated with the Cone Penetration Test (CPT) results, as proposed by Imai and Tonouchi (1982) and Poulos (1989, 1994):

$$E_s = 53q_c^{0.61} \quad [\text{MPa}] \tag{10.15}$$

where q_c is the cone resistance (in MPa).

It must be stated that the empirical correlations presented above (Equations (10.11)–(10.15)) can only be expected to provide an approximate estimate of initial soil modulus and may be rather inaccurate if applied to cases outside the scope of previous experience. Thus, as discussed by Gazetas (1991), their use may only be recommended in practice in some cases as follows: (a) in feasibility studies and preliminary design calculations; (b) for final design calculations in big projects as supplementary data or in small projects as main data; (c) for initial data in back analyses; (d) to provide an order-of-magnitude check against the experimentally determined values.

It should be emphasised that such correlations refer to the axial response of pile foundations. For piles under lateral loading, the effects of pile installation and pile–soil separation on the upper soil layer can have a significant influence on the values of soil stiffness, and hence the values of E_s adopted for the axial response may be reduced up to 50% or more.

As regards the soil Poisson's ratio, its effect is quite minor when the analysis is based on the use of Young's modulus rather than shear modulus. For saturated clays under undrained conditions, a value of 0.5 is relevant while, for most clays and sands, the drained value is usually in the range 0.3–0.4 (Poulos, 1994). Values of Poisson's ratio may also be approximated using the empirical formula (Duncan and Mokwa, 2001):

$$v_s = \frac{1 - \sin\phi}{2 - \sin\phi} \tag{10.16}$$

where the value of ϕ (friction angle) should be the total stress shear strength parameter ϕ_u for short-term undrained conditions and the effective stress shear strength parameter ϕ' for long-term drained conditions.

For a non-linear analysis, it is also necessary to assess the axial and lateral pile shaft resistance, and the end-bearing resistance, as discussed the earliersection on limiting pile-soil stress. Further information on this subject is provided in the work by Poulos (1989), Fleming *et al.* (1992) and Tomlinson (1994).

Applications and design analysis

Attention is turned to the application of available numerical analyses to practical problems involving real soils. Three published case histories are considered, involving single piles and pile groups subjected to either axial or lateral loading. In each case, the rationale for the selection of the soil parameters is described briefly, and then the predictions from selected methods of analysis are compared with the field measurements.

North London railway viaduct

Before proceeding to the analysis of the case histories, it is found instructive to discuss the results obtained from different numerical codes in the analysis of a 3 × 3 pile group subjected to a combination of vertical loads, horizontal loads and moments and embedded in London Clay. This project was part of the foundation design of a high-speed railway viaduct in North London. The bored cast-in-situ reinforced concrete piles are 17 m long, 0.9 m in diameter, with a centre-to-centre spacing of three pile diameters, and with the underside of the pile cap assumed at the top of the London Clay. The assumed Young's modulus for the piles is 25 GPa. A profile of undrained shear strength (C_u) of 50 + 9.4z kPa has been adopted, where z is the depth in m below the top of the London Clay. An adhesion factor of 0.6 is employed, while the hyperbolic curve fitting constants have been taken as 0.0 and 1.0 for the axial response of the shaft and the base, respectively, and 0.9 for the lateral response.

For the axial response, the profile of soil modulus has been derived from the correlation $E_s = 400C_u$ for the linear analyses and from $E_s = 1500C_u$ for the non-linear analysis. For the lateral response, the profile of soil modulus has been assumed to increase linearly with depth from a value of zero at the top of the London Clay (conservatively) at a rate of 4.14 MPa/m for the linear analyses and 6.15 MPa/m for the non-linear analysis. The soil Poisson's ratio has been taken as 0.5.

The applied vertical loads (V) result from the combined effect of live and dead loads, whereas the horizontal loads (H) and moments (M) are generated by the high-speed trains braking and accelerating. For the load case presented herein, the loads acting on the cap have been estimated as V = 14200 kN, H = 470 kN and M = 3200 kNm.

This problem has been analysed using the computer programs MPILE, DEFPIG and PGROUPN (both the linear and non-linear versions). Table 10.7 summarises the main results obtained from the analyses. In the linear range, there is a reasonably good agreement between the group deformations and axial load distribution predicted by the different codes. However, it is important to note the significant differences between the predictions of the pile head lateral loads and bending moments. As discussed previously, due to the interaction between the axial and lateral responses of the piles, higher loads are expected to occur for the piles in the leading row than for the piles in the trailing row of the group. While this load-deformation coupling effect is modelled by the PGROUPN analysis, MPILE and DEFPIG disregard the interaction between the axial and lateral responses and therefore predict the same lateral loads and bending moments for both the leading and trailing rows of the group. This results in a significant underestimate of the maximum values of lateral load and bending moment and hence may lead to an unsafe design of the piles.

If the effects of soil non-linearity are accounted for by means of the PGROUPN analysis, two main features of behaviour are observed:

Table 10.7 Comparison of alternative numerical analyses for a railway viaduct in North London

	MPILE	*DEFPIG*	*PGROUPN (Linear)*	*PGROUPN (Non-linear)*
Group centre settlement (mm)	9.0	11.3	11.6	4.0
Group deflection (mm)	3.2	4.3	3.9	2.7
Axial load at corner piles of leading row (kN)	2220	2210	2230	2100
Axial load at corner piles of trailing row (kN)	1700	1670	1640	1520
Lateral load at corner piles of leading row (kN)	66	62	94	76
Lateral load at corner piles of trailing row (kN)	66	62	23	35
Bending moment at corner piles of leading row (kNm)	120	177	225	179
Bending moment at corner piles of trailing row (kNm)	120	177	87	124

1 A prediction of lower (and more realistic) group deformations.
2 A decrease of predicted loads on the most heavily loaded row of piles (i.e. the leading row) and hence a more uniform load distribution between the piles.

It should be emphasised that in this case, due to the low load level, the differences between the linear and non-linear PGROUPN results are mainly a consequence of the higher value of soil modulus adopted in the non-linear analysis (i.e. an initial value), rather than the effect of soil non-linearity.

This observation confirms the view already expressed by other authors (Randolph, 1994; Mandolini and Viggiani, 1997); at low load levels (and hence for a high safety factor), soil non-linearity has a relatively small effect on pile group response, provided that the group response is calculated using the initial value of soil modulus. However, when the factor of safety is low, consideration of soil non-linearity becomes essential. It is hoped that the above philosophy will find a wider application in design practice.

Comparison with field test data by O'Neill et al. *(1982)*

O'Neill *et al.* (1982) reported the results of axial loading tests on single piles and pile groups driven into a stiff overconsolidated clay at a site located in Houston. The piles were closed end tubular steel pipes with Young's modulus of 210 GPa, external diameter 274 mm, wall thickness 9.3 mm and a penetration depth of 13.1 m. The group piles were connected by a rigid cap with a clearance of 0.9 m from the groundline and were arranged in a 3 × 3 configuration with centre-to-centre spacing of three pile diameters. The soil parameters adopted for the PGROUPN non-linear analysis are based on the data summarised by Poulos (1989) in his Rankine Lecture, i.e. a profile of the initial soil modulus of 100 MPa at ground level, increasing linearly to 400 MPa at the pile base level (as deduced from seismic cross-hole data), and a profile of undrained shear strength of 40 kPa at the surface, increasing linearly to 175 kPa at the level of the pile base (as deduced from laboratory triaxial tests). The soil Poisson's ratio and the adhesion factor have been taken as 0.5, while the hyperbolic curve fitting constants have been assumed to be 0.0 for the shaft and 1.0 for the base.

Figures 10.9 and 10.10 show a favourable agreement between the computed and measured load–settlement behaviour of the single pile and the 9-pile group. The results show that the initial tangent soil modulus, as derived from seismic cross-hole data, may be successfully used in the prediction of the pile settlement, thereby confirming the findings of Mandolini and Viggiani (1997). It is worth noting that the adopted cross-hole profile of initial soil modulus corresponds to that which would have been derived from a correlation $E_s = 2500C_u$.

Figures 10.9 and 10.10 also show the load–settlement curves obtained by the hybrid method by Chow (1986a) in which the individual pile response is modelled using the load-transfer method and the interaction between piles is effected using a BEM approach based on Mindlin's solution. It is important to note that the results obtained by Chow for the pile group have been based on parameters derived from back-analysis of single-pile test data, whereas the PGROUPN results have been obtained using soil parameters directly derived from the site investigation

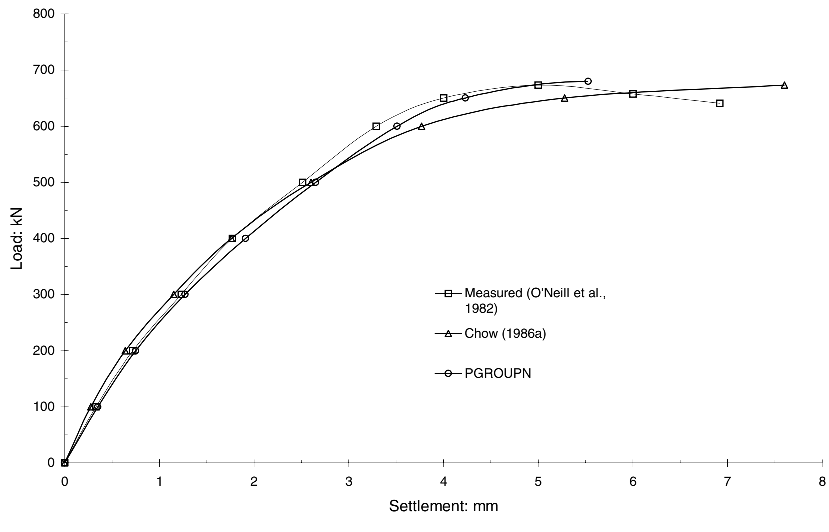


Figure 10.9 Comparison of load-settlement response for single pile.

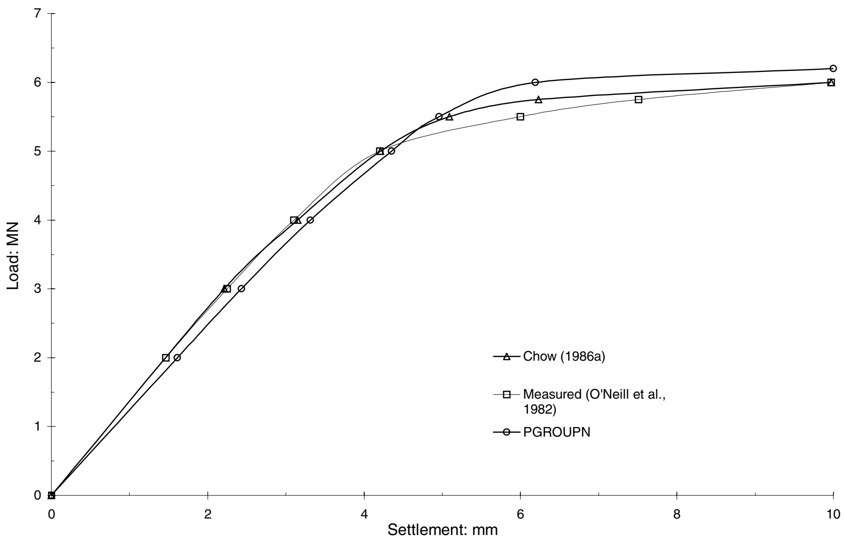


Figure 10.10 Comparison of load-settlement response for 9-pile group.

data. This confirms the usefulness of the PGROUPN approach for practical problems, particularly when no pile test results are still available.

Figures 10.11 and 10.12 report the computed and measured axial load distribution with depth among the piles in the 9-pile group at a working load of 2.58 MN and at a load nearing failure of 5.66 MN, respectively. In addition, Table 10.8 shows the computed and measured axial loads taken by the individual pile heads under the group loads mentioned above. The results also include the predictions obtained from MPILE and the linear version of PGROUPN using a secant value of the soil modulus based on the correlation $E_s = 500C_u$.

It is worth noting that, even at a working load level, the linear solutions overestimate the load taken by the corner pile. Closer to the failure load of the group, the effect of non-linearity is to cause a redistribution of the loads in the individual piles (i.e. the share of load carried by the corner piles progressively decreases and that of the central pile increases), leading to a more uniform distribution. It is clear that, at this load level, the degree of accuracy of the analysis would to a large extent depend on the agreement between the assumed ultimate pile capacities and the actual values in the field. For instance, O'Neill and colleagues report that the centre pile carried the highest load at failure, as contrasted to the lowest at working load, due to a slightly higher end-bearing load that may have resulted from higher effective confining stresses in the soil in the interior of the group. It should be emphasised that, at this load level, the linear analyses are not strictly applicable, but the actual trend is well reflected in the non-linear solutions.

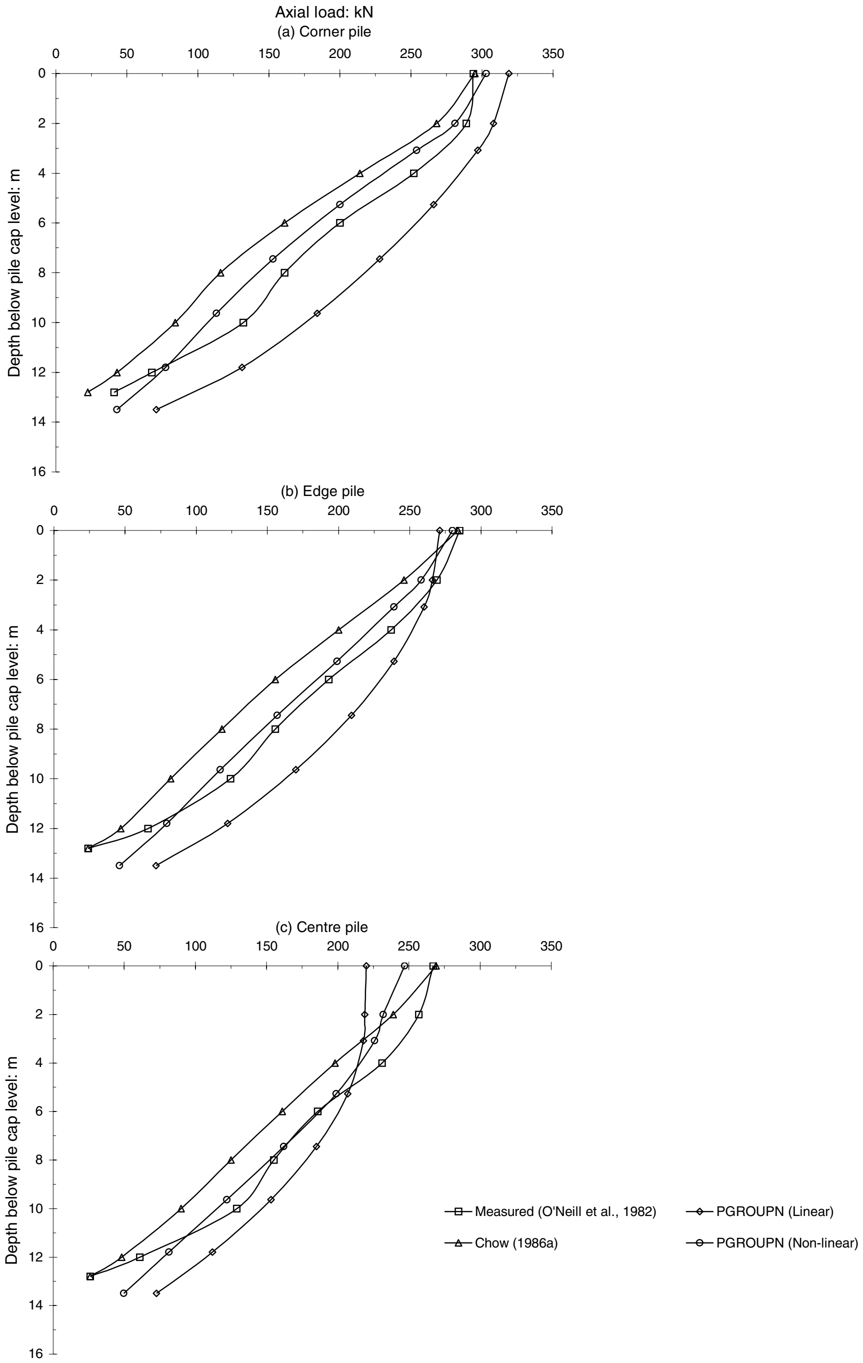


Figure 10.11 Comparison of axial load distribution in 9-pile group at a working group load of 2.58 MN (Note: Curves by Chow (1986a) were reported in Chow (1986b)).

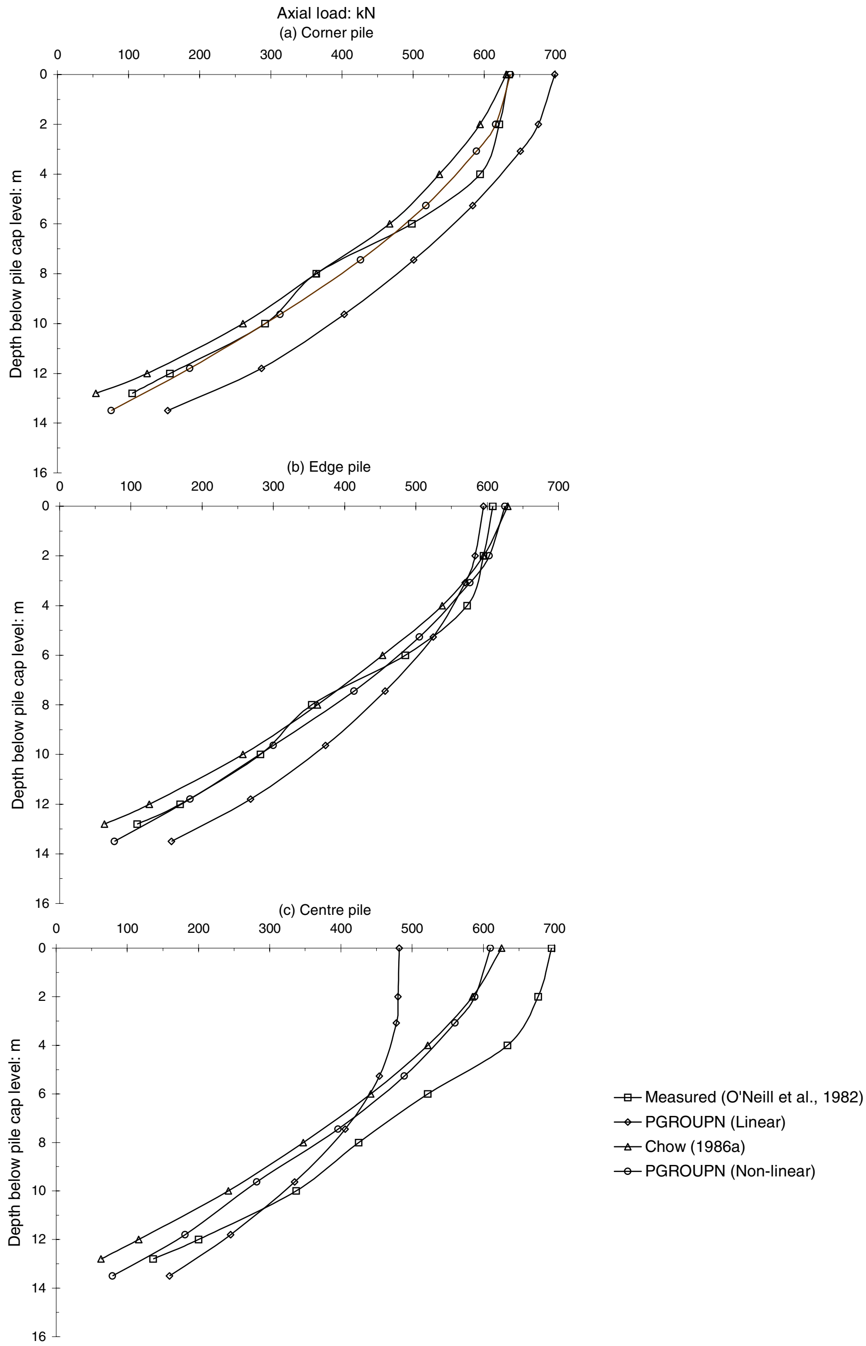


Figure 10.12 Comparison of axial load distribution in 9-pile group at a group load nearing failure of 5.66 MN (Note: Curves by Chow (1986a) were reported in Chow (1986b)).

Table 10.8 Comparison of axial load distribution to individual pile heads in 9-pile group

Method	*Average pile loads (kN)*					
	Total load = 2.58 MN			*Total load = 5.66 MN*		
	Corner	*Edge*	*Centre*	*Corner*	*Edge*	*Centre*
Measured (O'Neill *et al.*, 1982)	294	285	267	635	608	696
PGROUPN (Non-linear)	303	280	247	637	625	610
Chow (1986a)	295	284	269	631	629	626
PGROUPN (Linear)	319	271	220	699	595	482
MPILE	315	273	226	692	599	496

Comparison with field test data by Briaud et al. *(1989)*

Briaud *et al.* (1989) described the results of axial loading tests on a single pile and a 5-pile group which were driven to failure in a medium dense sand at a site located in San Francisco. The piles were tubular steel pipes with Young's modulus of 160 GPa, external diameter 273 mm, wall thickness 9.3 mm, driven to a depth of 9.15 m through a 300 mm diameter hole predrilled to a depth of 1.4 m. The single pile was loaded at 1.5 m above the groundline. The group piles were arranged in the configuration shown in the inset to Figure 10.14, and connected by a rigid cap with a clearance of 0.6 m from the groundline. The soil profile consists of a hydraulic fill made of clean sand, about 11 m thick, overlain by 1.4 m of sandy gravel and underlain by sand interbedded with layers of stiff silty clay down to the bedrock found at a depth of 14.3 m below ground level. The water table is 2.4 m deep.

The soil parameters adopted for the PGROUPN analysis are based on a subsoil idealisation with two layers resting on a rigid base: for the lower soil layer (2.4–14.3 m), a profile of the initial tangent soil modulus of 138 MPa at a depth of 2.4 m, increasing linearly at the rate of 4.6 MPa/m (as deduced from the CPT profile using Equation (10.15)), a Poisson's ratio of 0.3 (from Equation (10.16)), a buoyant unit weight of 10.1 kN/m^3 and a friction angle of 35.4° (from the soil investigation). The pile–soil interface angle may generally be taken as 5 degrees less than the friction angle (Reese and Van Impe, 2001), and the coefficient of horizontal soil stress (K_s) equal to 1.2 (Fleming *et al*., 1992). For the upper soil layer (0–2.4 m), a constant value of soil modulus equal to 138 MPa has been adopted (it should be noted that the predrilled hole disconnects the piles from the top 1.4 m of gravelly soil). The remaining parameters are the same as those for the underlying layer, with the exception of the soil unit weight which is equal to the dry value, 15.7 kN/m^3. The hyperbolic curve fitting constants for the analysis have been assumed to be 0.5 for the shaft and 1.0 for the base.

Figures 10.13 and 10.14 show a favourable agreement between the computed and measured load–settlement behaviour of the single pile and the 5-pile group. It

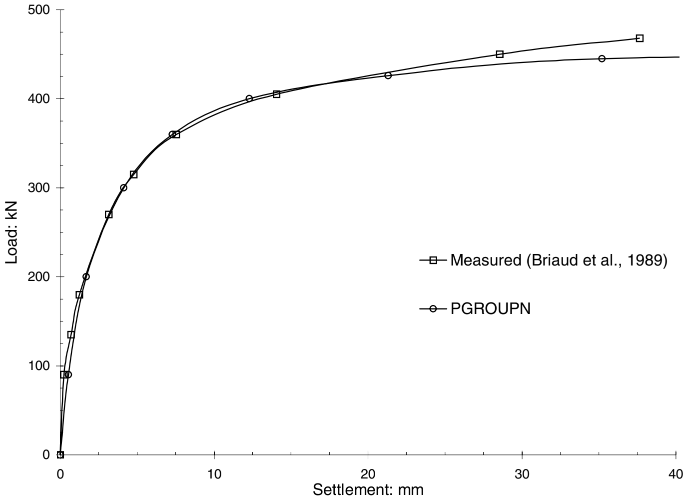


Figure 10.13 Comparison of load-settlement response for single pile.

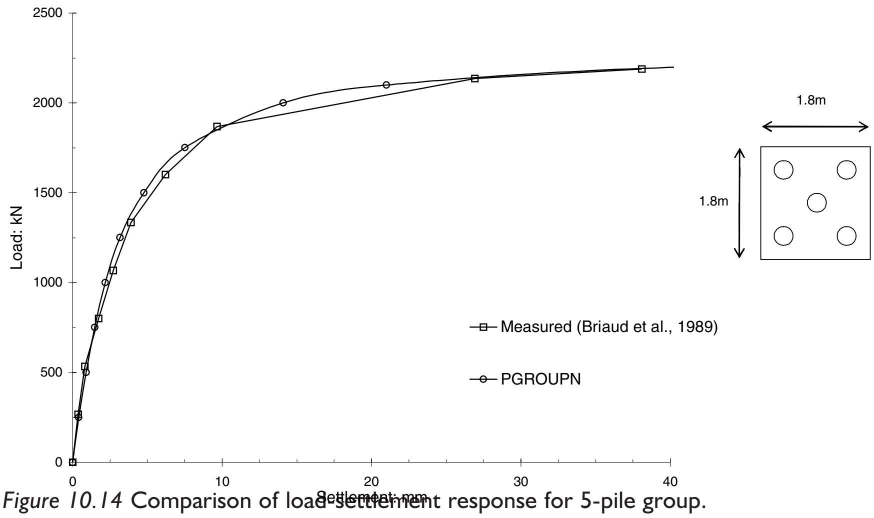


Figure 10.14 Comparison of load-settlement response for 5-pile group.

is worth noting that the measured ultimate capacity for the single pile was 505 kN while that for the pile group was 2499 kN, thereby giving a group efficiency of 0.99. Thus, in this case, no increase in pile shaft capacity due to the effects of driving neighbouring piles has taken place.

Comparison with field test data by Huang et al. *(2001)*

As part of the design of the high-speed rail system in Taiwan, Huang *et al.* (2001) reported the results of lateral load tests on single piles and pile groups installed at a site located in Taipao Township. The bored cast-in-situ reinforced concrete piles were 34.9 m long, 1.5 m in diameter, with a Young's modulus of 27.6 GPa. The group piles were connected by a massive reinforced concrete cap and arranged in a 2 × 3 configuration with centre-to-centre spacing of three pile diameters, as shown in the inset to Figure 10.16. The lateral load was applied at the level of the ground surface for both the single pile and the pile group. The soil was generally classified as silty sand or silt with occasional layers of silty clay. The water table is at approximately 1 m below the ground surface.

The soil parameters adopted for the PGROUPN analysis include a profile of the initial soil modulus of 77 MPa at a depth of 1 m (where the bottom of the pile cap was located), increasing linearly at the rate of 9.5 MPa/m, as deduced from the seismic cone penetration test (SCPT) shear wave velocity measurements using a Poisson's ratio of 0.35 (from Equation (10.16)). Based on the soil stratification derived from CPT, and for the purpose of evaluating the response to lateral loading (for which the soil properties in the top eight pile diameters are most relevant), it is reasonable to idealise the soil profile as a single cohesionless layer with a friction angle of 30°. This has been derived from the widely adopted correlation with standard penetration test (SPT) data reported in Tomlinson (1995), using an N value of 10 for the soil in the top eight diameters. Other input parameters for the PGROUPN analysis include a pile–soil interface angle of 25° (i.e. 5 degrees less than the friction angle), a buoyant unit weight of 10 kN/m^3 (assumed), and a coefficient of horizontal soil stress (K_s) of 0.7 (Fleming *et al*., 1992). The hyperbolic curve fitting constants have been taken as 0.5 and 1.0 for the axial response of the shaft and the base, respectively, and 0.9 for the lateral response (it should be noted that the value of the hyperbolic constants for the axial response has in effect no influence on the lateral response of the group).

Figures 10.15 and 10.16 report the computed and measured pile head load–deflection response of the single pile and the 6-pile group, respectively. The agreement for the single pile results is favourable, whereas, for the 6-pile group, the deflections computed by PGROUPN are slightly overestimated. These differences may partially be explained disregarding any shear resistance that might have developed along the base of the massive cap. In addition, other factors such as the cracking of the pile section and the rigidity of the connection of pile to pile cap can influence the lateral group response, particularly under large loads. These factors are not readily modelled in the PGROUPN analysis.

Figures 10.15 and 10.16 also report the results obtained by Huang and colleagues using the computer program GROUP, based on the use of *p-y* curves. They found that none of the *p-y* curves derived from the soil tests dilatometer test (DMT) yielded reasonable predictions of pile deflection profiles of the single pile and the pile group. The *p-y* curves were then adjusted until a good match between the measured and computed load–deflection profiles was achieved.

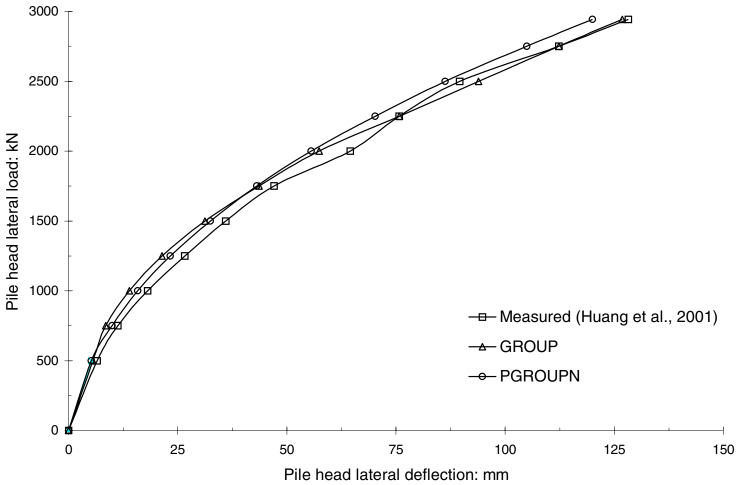


Figure.10.15 Comparison of load-deflection response for single pile.

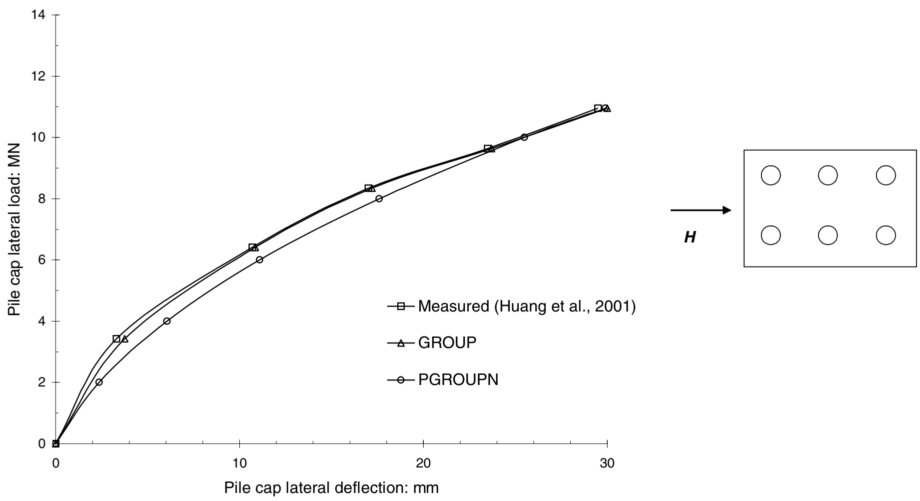


Figure 10.16 Comparison of load-deflection response for 6-pile group.

A comparison between the bending moment profiles predicted by PGROUPN and GROUP for the single pile and the 6-pile group is presented in Figures 10.17 and 10.18, respectively, showing a reasonable agreement between the analyses.

Overall, it may be concluded that the PGROUPN results are of comparable accuracy to those obtained from GROUP. However, it should be emphasised that the PGROUPN analysis is based on the assessment of intrinsic soil properties determined from the soil investigation, whereas the GROUP analysis made use of backfigured data from loading tests on the single pile and the pile group.

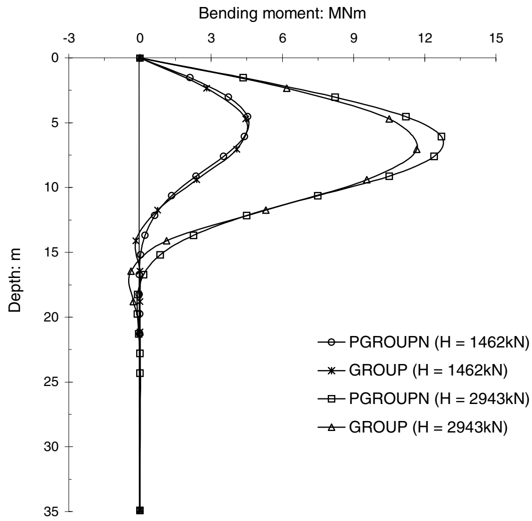


Figure 10.17 Comparison of moment profiles of single pile.

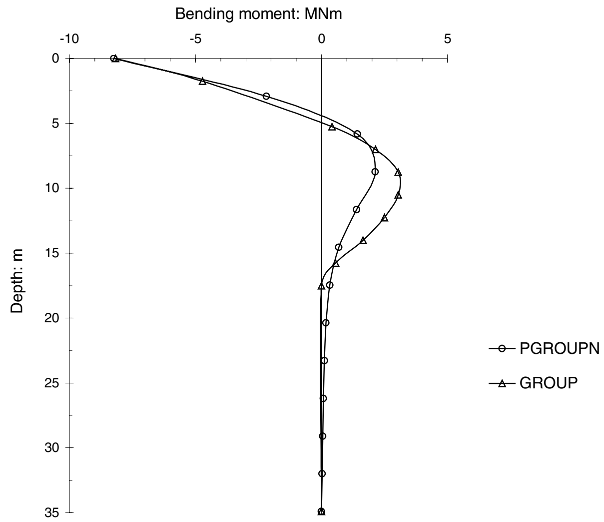


Figure 10.18 Comparison of moment profiles of leading row of piles in 6-pile group under the maximum applied lateral load $H = 10{,}948$ kN.

Final remarks

As early as 1977, Burland, Broms and de Mello pointed out that the primary reason for inclusion of piles as part of a foundation system is to satisfy a serviceability limit on deformations; nevertheless, traditionally pile designers have asked themselves how many piles are needed to carry the weight of the building rather than asking themselves the question of how many piles are needed to reduce settlements to an acceptable level.

In spite of this primary purpose of piles, twenty-five years later, common practice in pile group design still concentrates on providing suitable capacity from the piles to carry the structural load, and estimation of the settlement is generally treated as a secondary issue. The dominance of capacity-based design, which is evident in current revisions of national and regional design codes, may partially be attributed to the common belief that predicting deformations is more difficult and less reliable than predicting capacity. In reality, however, the reverse is often true for pile foundations (Randolph, 1994; Mandolini and Viggiani, 1997).

Thus, provided there is a minimum factor of safety, which may be as low as 1.5, pile group design should be approached in terms of satisfying the settlement criterion, rather than being based on a notional factoring of the ultimate state of each pile (Fleming *et al*., 1992). If this design philosophy is adopted, and hence low safety factors are employed, consideration of non-linear soil behaviour becomes essential. This would result in an improved understanding of pile group behaviour and hence in more effective design techniques.

In this chapter, the effects of soil non-linearity on pile group response, as measured experimentally and as predicted by current numerical analyses, have been discussed. A computer program, called PGROUPN, for pile group analysis and design has been presented. It has been shown that the proposed method, by taking into account the continuous nature of pile–soil interaction, removes the uncertainty of empirical *t-z* and *p-y* approaches and provides a simple design tool based on conventional soil parameters.

Use of the program may lead to a number of significant advantages in practice. For example, even for a purely linear analysis, the PGROUPN solution is capable of modelling important features of group behaviour which are normally disregarded by the other numerical approaches. Consideration of such features is essential in order to obtain a more realistic prediction of the load distribution between the individual piles of the group.

Another significant aspect of group behaviour which is not treated adequately by the other numerical procedures is the effect of soil non-linearity. The main advantage of a non-linear group analysis system over a linear one is that it has the desirable effect of demonstrating a reduction of the corner loads in large groups in both the horizontal and vertical senses. It has been shown that, even at typical working load levels, this reduction is significant. These observations are of basic importance in practice and may lead to tangible improvements in design procedures and worthwhile savings in construction costs.

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