

# INFLUENCE OF CAP RIGIDITY ON PILE GROUP DESIGN

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**ABSTRACT** Piles in a group are often connected through a cap which has a relatively small rigidity. In that case, the assumption of fully rigid cap adopted by some pile designers is misleading and can lead to an underestimate of pile deformations and an overestimate of forces and moments in the outer piles of the group, especially for larger groups. This may partially be due to a lack of readily available numerical codes, given that commercial software for pile group design is mainly based on the rigid cap assumption (e.g. Repute, PIGLET, DEFPIG, GROUP). In an attempt to overcome this limitation, this paper presents an extension of the Repute analysis to the fully-flexible cap (i.e. individual piles not connected by cap), a feature which is particularly useful in the design of large pile groups. Validity of the approach is assessed through comparison with alternative numerical analyses and a case history. In addition, a database of real cases is collated showing the relationship between the raft geometry and rigidity. Results provide guidance on the field of applicability of the rigid and flexible cap assumptions, indicating that most large rafts are likely to belong to the perfectly flexible category.

## 1. Introduction

A key aspect in the design of pile groups is the proper evaluation of the cap stiffness. For pile groups of small to medium size (say up to 25 piles), the assumption of rigid cap is generally reasonable for practical purposes. However, as the pile-group size increases, the cap stiffness is reduced and the following aspects need to be considered:

- increase of differential settlements and angular distortion across the pile group;
- more uniform load and moment distribution among the group piles.

In current design practice, however, the above issue is not always correctly addressed and the assumption of fully-rigid cap is rather indiscriminately adopted. This may partially be due to a lack of readily available numerical codes, given that commercial software for pile group design is mainly restricted to the rigid cap assumption, such as the Winkler-based approach adopted by GROUP (Reese *et al.*, 2016), and the boundary element analyses used by Repute (Bond and Basile 2018), PIGLET (Randolph, 2004), and DEFPIG (Poulos, 1990). More sophisticated numerical analyses are also available, generally based on the finite element (e.g. Plaxis 3D) or the finite difference (e.g. Flac-3D) methods. However, due to their complexity and high running costs (both in terms of computational time and efforts in data handling), these programs are not readily applicable to routine pile-group design and their use is confined to special cases (e.g. large piled rafts).

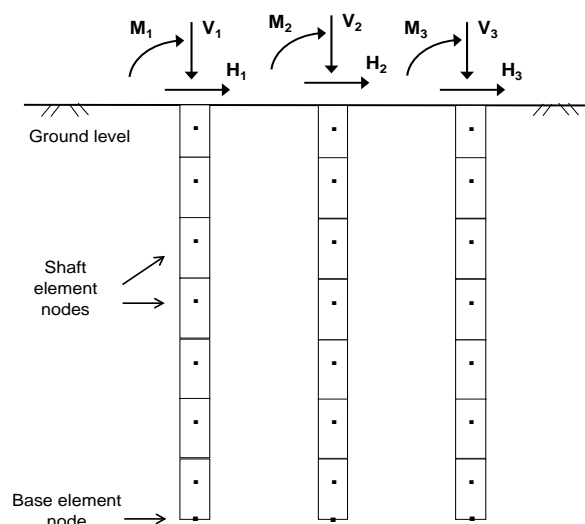
In an attempt to overcome the above limitation, this paper presents an extension of the Repute analysis (currently restricted to fully-rigid caps) to the case of a fully-flexible cap, i.e. individual piles not connected by cap. Accuracy of the proposed analysis is verified through comparison with alternative numerical solutions and a case history. In addition, in order to show the relationship between the raft geometry and rigidity, a number of well-documented real cases is examined.

## 2. Analysis method

The proposed analysis method is based on the 3D boundary element (BEM) solution implemented in the code PGROUPN, i.e. the calculation engine of Repute software. The analysis takes into account the simultaneous influence between all elements of all piles in the group, i.e. a "complete" solution of the soil continuum is provided. Pile-group effects are therefore evaluated as a matter of course, thereby overcoming the approximations of traditional interaction factor approaches and the fundamental limitations of Winkler models (based on empirical multipliers to account for group action). In addition, by retaining soil continuity, the input soil parameters have a clear physical meaning (e.g. the soil Young's modulus and strength properties) and can be measured directly in soils investigation.

A detailed description of the theoretical formulation adopted in PGROUPN is reported in Basile (2003) and, hence, only a brief outline of the method is given below. Following the typical BEM scheme, the pile-soil interface is discretized into a number of cylindrical elements, while the base is represented by a circular (disc) element, as illustrated in Figure 1.

Figure 1 BEM schematisation of the problem



The analysis is based on a sub-structuring technique in which the piles and the surrounding soil (modelled using the well established solution of Mindlin, 1936) are considered separately and then coupled by imposing the compatibility and equilibrium conditions at the pile-soil interface. Given unit boundary conditions, the pile and soil domains are solved, thereby leading to the distribution of displacements, stress, forces, and moments in the piles under the external loads. General 3D loading conditions (axial, lateral, and moments) on each pile of the group can be examined.

Non-linear soil response is modelled by adopting the common Duncan-Chang hyperbolic stress-strain model within a stepwise incremental procedure. The external loads on the piles are applied incrementally and, at each increment, a check is made that the stress state at the pile-soil interface does not violate the yield criteria. This is achieved by specifying limiting values at the pile-soil interface according to the classical equations for the ultimate shaft and base resistance (e.g. Basile, 2003). The elements of the pile-soil interface which have yielded can take no additional load and any increase in load is therefore redistributed between the remaining elastic elements until all elements have failed.

### 3. Influence of cap rigidity

In practical applications, a simple check on the assumption of cap rigidity may be performed by calculating the raft-soil stiffness ratio ( $K_{rs}$ ) as defined by Horikoshi and Randolph (1997):

$$K_{rs} = 5.57 \frac{E_r}{E_s} \frac{1-\nu_r^2}{1-\nu_s^2} \left(\frac{B_r}{L_r}\right)^{0.5} \left(\frac{t_r}{L_r}\right)^3 \quad (1)$$

where the subscripts  $r$  and  $s$  denote the raft and soil properties, respectively,  $E$  is the Young's modulus,  $\nu$  is the Poisson's ratio,  $B_r$  is the raft width,  $L_r$  is the raft length (with

$B_r \leq L_r$ ), and  $t_r$  is the raft thickness. In the case of layered soil profiles,  $E_s$  is the representative soil Young's modulus beneath the raft and may be taken as the weighted average value over a representative depth equal to the raft equivalent radius, i.e.:

$$E_s = \frac{\sum_{i=1}^n h_i E_{si}}{\sum_{i=1}^n h_i} = \frac{\sum_{i=1}^n h_i E_{si}}{\sqrt{B_r L_r / \pi}} \quad (2)$$

where  $h_i$  is the thickness of layer  $i$ ,  $E_{si}$  is the soil Young's modulus of layer  $i$ ,  $n$  is the number of soil layers within the representative depth, and  $\sqrt{B_r L_r / \pi}$  is the raft equivalent radius. In the selection of the  $E_s$  value to be used in Equ. (1), it is also important to account for the appropriate strain level applied to the soil by the foundation. As an indication, following O'Brien (2012), a secant value of  $E_s$  equal to 70% of the small strain value may generally be adopted for pile groups designed with a typical factor of safety of about two.

The following limits for  $K_{rs}$  values resulting from Equ. (1) are recommended, as illustrated in Figure 2:

- For  $K_{rs} < 0.01$ , the raft behaves as "fully flexible", i.e. piles can be considered as not connected by cap (and hence interacting with each other only through the soil). In this case, the raft is unable to redistribute external load between the piles and hence the loads acting on each pile are equal to the actual loads applied on that pile. Assessment of differential settlements (and angular distortion) across the pile group is a key design issue. For design purposes, a higher limit for  $K_{rs}$  may be adopted and rafts with  $K_{rs} < 0.1$  can generally be considered as "practically flexible".
- For  $0.1 < K_{rs} < 1.5$ , the raft has an "intermediate" stiffness which should properly be considered in design via a suitable numerical analysis, given that the raft-soil interaction will have a dominant role on raft behaviour.

Figure 2  $K_{rs}$  database and relationship with  $t_r/D_{equ}$

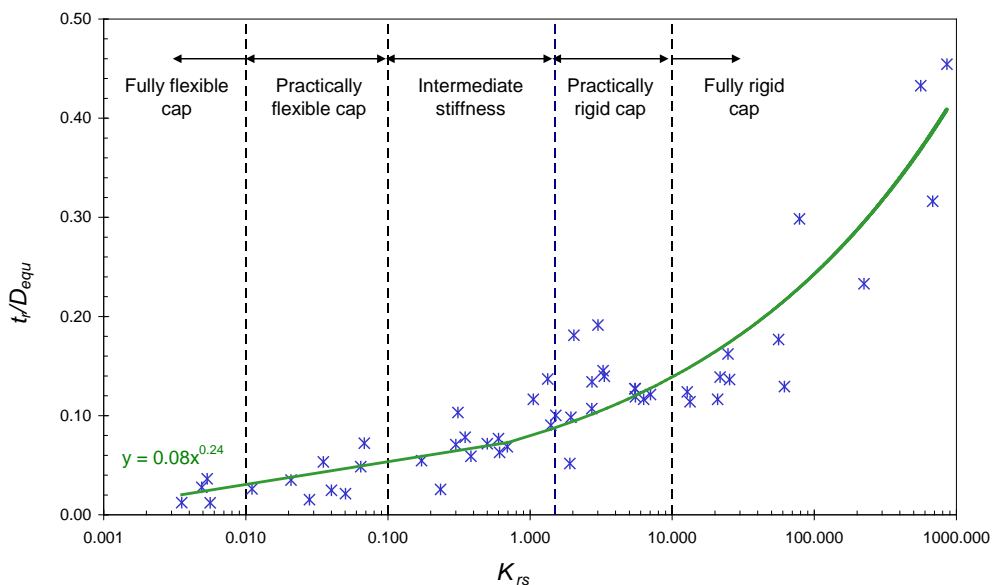


Table 1 Details of real cases considered in  $K_{rs}$  database

Case	Reference	Description	$n$	$L_p$ (m)	$s/D$	$L_r$ (m)	$B_r$ (m)	$D_r$ (m)	$t_r$ (m)	$t_r/D_{equ}$	$E_s$ (MPa)	$K_{rs}$
1	Author's database	Viaduct	20	36.0	3.0	15.5	13.2	-	2.00	0.12	23	12.8
2	Author's database	Viaduct	12	22.1	3.4	19.5	5.1	-	1.50	0.13	14	2.72
3	Author's database	Building	220	18.0	6.1	42.6	22.3	-	0.91	0.03	88	0.01
4	Author's database	Building	18	13.0	5.0	25.1	6.3	-	0.76	0.05	57	0.04
5	Author's database	Building	159	31.0	2.5	54.1	34.1	-	3.50	0.07	600	0.07
6	Author's database	Viaduct	10	15.1	5.2	6.3	1.4	-	1.00	0.30	4	78.8
7	Author's database	Building	36	15.0	-	19.8	9.8	-	2.00	0.13	26	5.49
8	Author's database	Tank	40	9.0	-	-	-	20.6	2.40	0.12	31	20.9
9	Author's database	Viaduct	8	16.0	3.1	6.2	2.2	-	0.80	0.19	50	3.01
10	Author's database	Wind turbine	26	7.5	-	-	-	17.2	2.50	0.15	106	3.27
11	Author's database	Bridge	4	5.0	2.5	2.7	2.7	-	1.40	0.45	26	859
12	Author's database	Building	45	10.0	3.1	24.0	12.0	-	1.50	0.08	83	0.35
13	Author's database	Wind turbine	16	25.0	-	-	-	17.5	1.20	0.07	92	0.69
14	Author's database	Building	35	25.0	3.0	5.9	4.1	-	0.90	0.16	21	24.7
15	Badellas et al. (1988)	Tank	112	40.7	3.2	-	-	37.6	0.80	0.02	35	0.05
16	Rampello et al. (2004)	Chimney	74	56.7	2.7	-	-	30.5	3.00	0.10	90	1.94
17	Liew et al. (2002)	Tank	137	27.9	4.3	-	-	19.5	0.50	0.03	11	0.23
18	Borsetto et al. (1991)	Chimney	281	25.0	3.3	-	-	30.4	4.25	0.14	154	3.33
19	Author's database	Viaduct	20	23.0	3.0	17.2	12.9	-	2.00	0.12	38	5.51
20	Author's database	Building	288	17.3-37.0	2.9	64.6	46.4	-	3.00	0.05	220	0.06
21	Author's database	Building	15	9.0	2.6	14.2	5.2	-	1.00	0.10	80	0.31
22	Author's database	Building	93	23.0	3.0	10.4	8.0	-	1.20	0.12	175	1.05
23	Author's database	Bridge	30	15.0	3.0	20.0	6.0	-	1.50	0.12	6	7.04
24	Author's database	Building	36	15.0	2.5	19.8	9.8	-	2.00	0.13	26	5.46
25	Author's database	Wind turbine	40	8.5	-	-	-	20.6	2.40	0.12	34	6.30
26	Author's database	Wind turbine	38	16.7	-	-	-	16.4	2.90	0.18	21	56.1
27	Author's database	Bridge	5	18.9	1.5	11.1	6.6	-	2.25	0.23	5	224
28	Author's database	Building	12	10.0	3.8	12.8	7.3	-	1.50	0.14	166	1.33
29	Sommer et al. (1985)	Building	42	20.0	3.6	24.5	17.5	-	2.50	0.11	49	2.72
30	Tazoh et al. (1987)	Bridge	64	22.0	2.5	12.0	12.0	-	1.75	0.13	7	61.5
31	Reul & Randolph (2004)	Building	25	22.0	-	-	-	-	3.50	0.07	-	0.30
32	Reul & Randolph (2004)	Building	64	31.0	-	58.8	58.8	-	6.00	0.09	-	1.40
33	Reul & Randolph (2004)	Building	40	30.0	-	-	-	-	4.70	0.08	-	0.60
34	Reul & Randolph (2004)	Building	47	25.0	-	-	-	-	2.00	0.02	-	0.04
35	Reul & Randolph (2004)	Building	54	14.0	-	37.1	37.1	-	3.00	0.07	-	0.50
36	Author's database	Viaduct	11	19.0	3.2	13.8	8.4	-	2.20	0.18	287	2.04
37	Kakurai et al. (1987)	Tank	5	22.7	-	-	-	11.6	0.60	0.05	14	1.91
38	Author's database	Wind turbine	24	31.0	-	-	-	18.0	2.50	0.14	27	21.7
39	Van Impe et al. (2015)	Tank	422	21.5	4.8	-	-	48.8	0.60	0.01	105	0.004
40	Jendeby (1986)	Building	48	25.0	-	38.0	14.0	-	0.40	0.02	3	0.03
41	Pirrello & Poulos (2014)	Building	172	46.1-76.1	-	88.0	77.5	-	5.50	0.06	95	0.38
42	Author's database	Building	174	50.0	2.9	49.4	34.2	-	-0.27	0.05	109	0.17
43	Chow & Small (2008)	Tank	180	26.5	-	71.0	61.2	-	0.90	0.01	53	0.006
44	Cooke et al (1981)	Building	351	13.0	3.6	43.3	19.2	-	0.90	0.03	160	0.005
45	Author's database	Building	149	25.0	10.6	70.4	36.4	-	2.00	0.04	123	0.02

Note:  $n$  = No. of piles,  $L_p$  = pile length,  $s/D$  = pile spacing,  $L_r$  = raft length,  $B_r$  = raft width,  $D_r$  = raft diameter,  $t_r$  = raft thickness,  $D_{equ}$  = raft equivalent diameter,  $E_s$  = soil Young's modulus,  $K_{rs}$  = raft-soil stiffness ratio

- For  $K_{rs} > 10$ , the raft behaves as "fully rigid" and is therefore able to redistribute external load between the piles. In this case, the loads acting on each pile can be determined by means of pile-soil interaction analysis, while differential settlements across the pile group are negligible. Based on the consideration that the  $K_{rs}$  value of Equ. (1) does not include the additional stiffening contribution provided by the piles and by the superstructure, a lower limit for  $K_{rs}$  may be assumed for design purposes and rafts with  $K_{rs} > 1.5$  may be considered as "practically rigid" (Randolph, 2003).

The above definition of  $K_{rs}$  has been used to verify the raft rigidity in No. 52 real cases including both published case histories and projects from the author's database. The main features for each case are listed in Table 1, ranging from small pile groups for bridges to large piled rafts for high-rise buildings, so that they are believed to be representative of a broad range of practical problems.

The computed  $K_{rs}$  values for each case are plotted on a logarithmic scale in Figure 2, varying from values of about 0.004 to 1000 (the best-fit power curve of the data set is also shown). In addition, in order to provide a simple geometric parameter purely based on the raft dimensions and independent of the raft and soil Young's modulus, the ordinate axis reports the corresponding  $t_r/D_{equ}$  values (where  $D_{equ} = 2\sqrt{B_r L_r/\pi}$  is the raft equivalent diameter). It may be observed that, for  $t_r/D_{equ} > 0.10$ , the raft generally tends to behave as a rigid raft, while for  $t_r/D_{equ} < 0.05$ , the raft is likely to belong to the flexible category. The latter result confirms the findings of Poulos (2016), i.e. large rafts tend to behave as fully flexible rafts. The above limits for  $t_r/D_{equ}$  may therefore be used for a preliminary indication of the raft rigidity in the absence of any information on the raft and soil Young's moduli (which is required for a more rigorous assessment of the raft stiffness using Equ. 1).

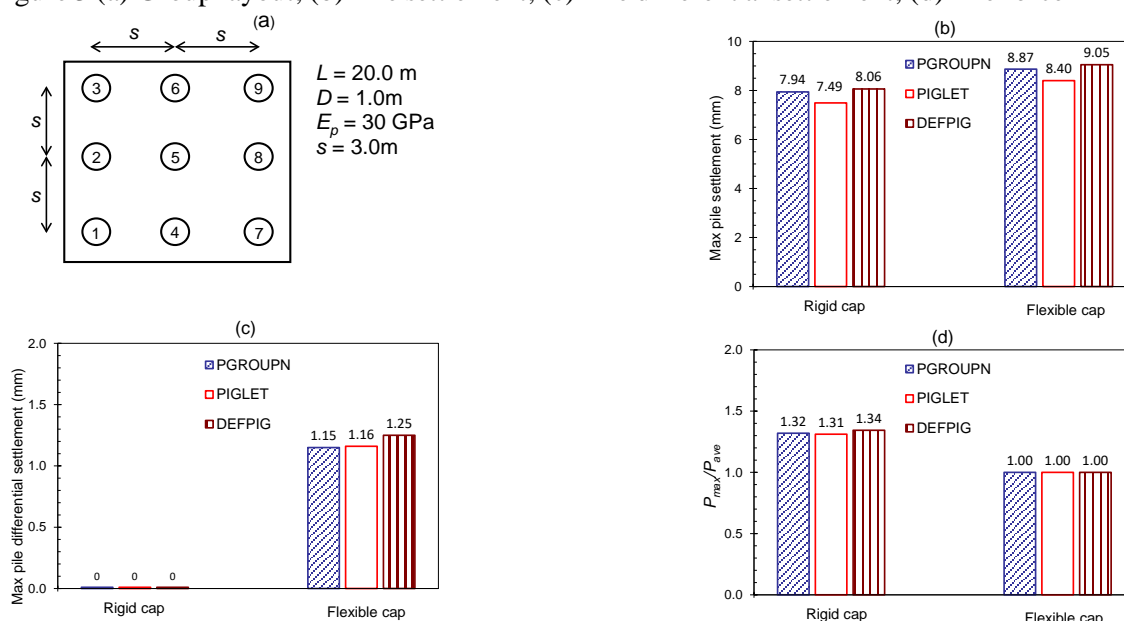
## 4. Numerical results

### 4.1 Comparison with PIGLET and DEFPIG

Accuracy of PGROUPN is assessed through a comparison with two established computer programs for pile-group analysis, i.e. PIGLET (Randolph, 2004) and DEFPIG (Poulos, 1990). A 9-pile group under vertical loading is analysed, as sketched in Figure 3a. The piles are embedded in a homogeneous elastic soil layer having a Young's modulus of 50 MPa and a Poisson's ratio of 0.5. In order to investigate the effect of pile cap rigidity, two limiting conditions are considered, i.e. a fully-rigid cap in which the total applied load is equal to 9 MN (i.e. an average of 1 MN/pile), and a fully-flexible cap in which the applied load on each pile head is equal to 1 MN. Results are reported in terms of the maximum values of pile settlement, differential settlement, and normalised force ( $P_{max}/P_{ave}$ ) in Figures 3b, 3c, and 3d, respectively, showing a good agreement between programs. From these figures, the following observations can be made:

- the fully-flexible cap results in a larger (by about 12%) value of maximum pile settlement as compared to the fully-rigid cap (clearly in the latter case the settlements of all piles are equal);
- the fully-flexible cap allows the evaluation of differential settlements across the pile group;
- in the case of fully-rigid cap, the cap redistributes external load between the piles, thereby resulting in a non-uniform load distribution, with the corner pile carrying the greatest proportion of load and the centre pile carrying least;
- for caps of intermediate stiffness, the two limiting conditions of the cap stiffness will provide lower and upper bounds for the maximum values of pile settlement, differential settlement and force.

Figure 3 (a) Group layout; (b) Pile settlement; (c) Pile differential settlement; (d) Pile force



## 5. Assessment of pile spring stiffnesses

Structural design software generally offers the facility to model the foundation system as a raft slab supported by springs representing the piles. The stiffness of these springs (defined as load divided by displacement) should be assessed by the geotechnical engineer via a pile-group analysis which takes into account essential aspects such as pile-soil-pile interaction and soil nonlinearity.

In order to determine the pile spring stiffness values, an iterative procedure between the structural analysis of the superstructure and the geotechnical pile-group analysis is commonly carried out. This requires the structural designer adjusting the spring stiffness values obtained from the geotechnical analysis until a reasonable match between calculated displacements of the structural and geotechnical analyses is obtained. The revised column loads are then provided back to the geotechnical engineer and the process repeated until an acceptable level of convergence is achieved (usually within a few iterations). Different spring stiffness values are computed for each component of load (i.e. vertical, lateral and moment); although attention is herein focussed on the vertical spring stiffness values, a similar approach can be adopted for assessing the lateral and rotational spring stiffnesses. However, vertical, lateral and moment loads should be considered separately in order to avoid unrealistic stiffness values that can arise under some combinations of vertical, lateral and moment loading.

It is worth noting that, in the assessment of pile spring stiffness values via a pile-group analysis, the assumption of fully-flexible cap is often a more reliable approach than that of fully-rigid cap (Poulos, 2017); thus, the proposed PGROUPN extension to the fully-flexible cap can have a useful role in design practice. In order to illustrate the application of the above approach, the 9-pile group previously described in Figure 3a with the ground profile reported in Table 2 is considered. Table 3 and Figures 4a-b report the vertical pile-head stiffness values (i.e. applied load divided by displacement) computed for the two limiting conditions of the cap stiffness. Considering that the ultimate pile capacity results in 8.6 MN, two loading conditions are selected, i.e. an

average serviceability (SLS) load of 3.5 MN/pile (i.e. a total load on the group of 31.5 MN) and an average ultimate (ULS) load of 5.0 MN/pile (i.e. a total load on the group of 45.0 MN). The following features of behaviour are observed:

- Different stiffness values are obtained for each pile location within the group, with the corner piles showing the largest stiffness and the centre pile (No. 5) the smallest one. This demonstrates the shortcomings of ignoring pile-soil-pile interaction effects and assuming constant spring stiffness values across the pile group (as incorrectly adopted by some structural engineers);
- The stiffness values for the fully-flexible cap assumption show a more uniform distribution across the pile group than those based on the fully-rigid cap assumption;
- The group stiffness values are significantly lower than the single-pile stiffness value, thereby confirming the importance of considering pile-soil-pile interaction effects;
- The stiffness values for the ULS case are significantly lower than those for the SLS case, thus attesting the value of accounting for soil nonlinearity effects.

Table 2 Ground model for non-linear analysis

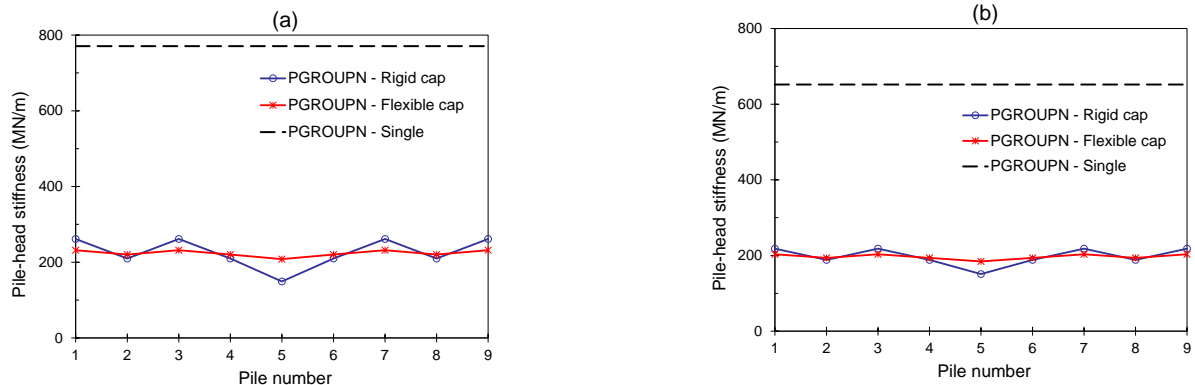
Depth range (m)	$E_s$ (MPa)	$\nu_s$	$f_s$ (kPa)	$f_b$ (MPa)
0-10	50	0.2	50	-
10+	200	0.2	100	5.0

Note:  $E_s$  = initial Young's modulus,  $\nu_s$  = Poisson's ratio,  $f_s$  = shaft friction,  $f_b$  = ultimate base resistance

Table 3 Pile-group stiffness values

Pile location	Pile-head stiffness values (MN/m)			
	SLS case		ULS case	
	Rigid cap	Flexible cap	Rigid cap	Flexible cap
Corner	261	232	218	203
Mid-side	210	220	189	194
Centre	149	208	151	185

Figure 4 Pile-head stiffness values: (a) SLS case; (b) ULS case



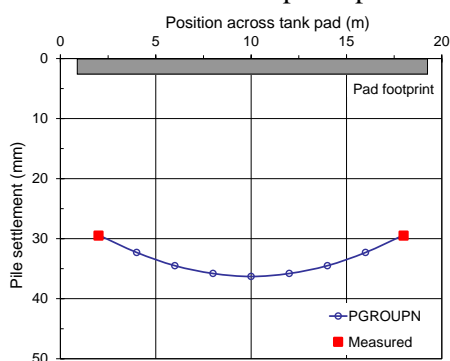


## 6. Case history

An example where the assumption of fully-flexible cap could be applied is that reported by Thorburn et al. (1983) for a molasses storage tank. The tank has a diameter of 12.5 m and is supported by a group of 55 precast concrete driven piles with a length of 29.0 m, a square section of 0.25 m, a Young's modulus of 26 GPa, and laid out on a triangular grid at a spacing of 2.0 m. The piles were not connected by a cap and a 2.0 m thick pad of dense granular material was constructed over the pile heads. Following Randolph (1994), the foundation soil can be modelled as a unique cohesive layer with parameters linearly increasing with depth, i.e. an undrained shear strength given by the expression  $C_u = 6 + 1.8z$  kPa and an initial value of soil Young's modulus ( $E_s$ ) based on a  $E_s/C_u$  ratio of 750. The soil Poisson's ratio is 0.5 while the average pile adhesion factor ( $\alpha$ ) can be calculated as 0.82 based on Poulos (2017).

Under a total applied load of 2,000 t (i.e. an average load of 357 kN/pile assuming uniform distribution among the 55 piles), measured settlements were in the range 29–30 mm for the piles located around the tank periphery, while the measured differential settlements across the pile group were less than 10 mm (i.e. within the accuracy of the instrumentation used to measure such settlements). The above measurements compare well with those predicted by PGROUPN which computes a settlement in the range 29–31 mm for the edge piles and a maximum differential settlement of 7 mm between the centre and edge of the pile group, as shown in Figure 5. Finally, it is noted that the computational time required for the 55-pile group analysis (using the non-linear soil model) is a matter of a few minutes on an ordinary computer, thereby confirming the value of PGROUPN as a practical numerical tool for pile group design.

Figure 5 Measured and computed pile settlements



## 7. Conclusions

The paper has described an efficient analysis method, based on the 3D BEM solution of the code PGROUPN (i.e. the calculation engine of Repute software), for determining the non-linear response of pile groups with a fully-flexible cap (i.e. individual piles not connected by cap). As compared to the analysis with fully-rigid cap, the proposed extension to the fully-flexible cap can be useful in practice given that:

- it allows computation of pile differential settlements and angular distortion across the pile group;
- it avoids overestimation of forces and moments in the outer piles of the group;
- it computes a larger (and hence more conservative) maximum pile settlement;
- it can be valuable in the assessment of pile spring stiffness values for the superstructure analysis.

The proposed method has been successfully compared with the results from alternative numerical solutions and field measurements which have confirmed the value of PGROUPN as a useful tool for pile group analysis. In addition, a database of real cases has been collated showing the relationship between the raft geometry and rigidity. Results provide insight into the field of applicability of the rigid and flexible cap assumptions, indicating that most large rafts are likely to belong to the fully-flexible category.

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