Kinematic Bending Moments in Pile Groups

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ABSTRACT:
The evaluation of kinematic bending moments developing in piles during earthquakes has been receiving increased interest from researchers. The significance of this aspect has also been recognized in recent seismic regulations such as Eurocode 8. Nevertheless, the kinematic loading effects do not usually receive proper attention by engineers and there is a need in industry to better understand this mode of response and to develop efficient methods for designing piles in seismic conditions. In an attempt to improve current understanding, a practical pseudostatic approach for estimating the deformation behaviour and internal forces of single piles and pile groups subjected to earthquake loading has recently been proposed by Basile (2010). In this paper, some further applications of the approach are presented, and attention is focused on the evaluation of kinematic bending moments in the piles.

Keywords: Pile, pile group, seismic, pseudostatic, inertial and kinematic effects

1. INTRODUCTION

Recent post-earthquake field investigations have demonstrated the significant role of kinematic effects in the development of pile damage (e.g. Nikolaou *et al*. 2001). Kinematic forces arise from the passage of seismic waves through the surrounding soil and become significant in the presence of strong discontinuities in stiffness of the soil profile. In this case, the large curvatures imposed to the piles by the vibrating soil in turn generate bending moments; these moments will develop even in the absence of a substructure and are referred to as "*kinematic*" moments, to be distinguished from moments generated by structural loading at the pile head ("*inertial*" moments).

The importance of kinematic effects has been recently recognized by seismic regulations such as Eurocode 8 (EN 1998-5, 2003) which states that "*piles shall be designed to resist the following two types of action effects: (a) inertia forces from the superstructure… ; (b) kinematic forces arising from the deformation of the surrounding soil due to the passage of seismic waves*", and that "*bending moments developing due to kinematic interaction shall be computed only when all of the following conditions occur simultaneously: (1) the ground profile is of type D, S_1 or S_2, and contains consecutive layers of sharply differing stiffness; (2) the zone is of moderate or high seismicity, i.e. the product a_gS exceeds 0.10g; (3) the supported structure is of class III or IV*".

While there is ample geotechnical experience on carrying out pseudostatic analyses for the inertial loading, no specific method of analysis is readily available to the designer in order to evaluate the pile response to kinematic loading. Although a number of analysis methods has been presented, ranging from simplified approaches to sophisticated dynamic boundary element (BEM) or finite element (FEM) formulations, there is a need to better understand the kinematic interaction effect and to develop efficient methods for predicting the pile behaviour in seismic conditions. A practical pseudostatic approach for estimating the deformation behaviour and internal forces of single piles and pile groups subjected to earthquake loading has recently been proposed by Basile (2010). The

approach is capable of accounting for both inertial and kinematic effects by means of a two-step procedure: (1) a free-field site response analysis is performed to obtain the maximum ground displacement profile caused by the earthquake; (2) a static analysis is carried out for the pile group, subjected to the maximum free-field ground displacement profile (kinematic loading) and to the static loading at the pile head based on the maximum surface acceleration (inertial loading). In this paper, validity of the approach is illustrated through comparison with alternative numerical analyses, and attention is focused on the critical evaluation of kinematic bending moments in the piles.

2. METHODS OF ANALYSIS

The complexity of the problem of seismic pile-soil interaction requires the use of computer-based methods of analysis. For both computational convenience and conceptual simplicity, the response of the complete soil-pile-superstructure system is generally computed using a substructure technique based on the superposition of kinematic and inertial response (Gazetas and Mylonakis, 1998). This can be achieved by following three interrelated analysis steps: (1) a free-field site response analysis is carried out to evaluate the response of the soil mass (in the absence of the piles) under seismic excitation; (2) a kinematic analysis is performed to assess the response of the piled foundation to the free-field incoming motion in the absence of inertial forces from the superstructure; (3) an inertial soil-structure interaction analysis is carried out to evaluate the dynamic response of the superstructure and the loads that this imposes on the foundation. The above decomposition of the problem does not necessarily imply that the three steps must be performed separately, and a *complete* (or *direct*) interaction analysis is, at least in principle, also possible. However, with foundations generally consisting of a group of piles, the complexity and computational cost of such analyses would be prohibitive for design, particularly when the effects of soil nonlinearity under seismic excitation become significant.

Numerical methods of analysis for estimating the deformation behaviour and internal forces of pile foundations under seismic excitation may be broadly classified into two categories: (1) Winkler-type approaches, and (2) continuum-based approaches, as described below.

2.1 Winkler model

This category is based on the so-called beam-on-dynamic-Winkler-foundation (BDWF) approach, in which the pile-soil interaction is simulated through a series of continuously distributed *springs* and *dashpots*, the frequency-dependent parameters of which (the dynamic stiffness "k" and the system damping "c") have generally been derived through calibration against results of rigorous continuum-based (FEM or BEM) dynamic analyses. While this approach has been used extensively to estimate the dynamic impedance of piles in relation to inertial interaction analyses, several studies have also employed the Winkler-type model to determine the kinematic response of piles (Kavvadas and Gazetas, 1993; Mylonakis *et al.*, 1997; Nikolaou *et al.*, 2001). In such studies, the springs and dashpots connect the pile to the free-field soil, with the wave-induced motion of the latter (computed with any available site-response analysis method) serving as the support excitation of the pile-soil system.

Based on the above methodology, some authors have proposed simplified closed-form expressions for estimating the kinematic pile bending moment at the interface between two soil layers (e.g. Dobry and O'Rourke, 1983; Nikolaou *et al.,* 2001; Mylonakis, 2001). However, such simplified expressions are more suitable for the preliminary design stages as they suffer from some significant limitations, i.e. the general overconservatism, the lack of any information on the pile-head moment (which is additional to the inertial pile-head moment), the limitation to a maximum of two soil layers, and the common assumption of "*thick*" soil layers (i.e. layers with thickness greater than the active pile length, which is typically of the order of 10 to 15 pile diameters below the ground surface).

Although Winkler models have become popular for the seismic analysis of pile foundations, mainly

due to their relative simplicity, one should be aware of the limitations associated with the approach:

(a) *Single-pile response*
   The Winkler model is of semi-empirical nature in that the spring coefficient is not a fundamental soil parameter but instead gives the overall effect of the soil continuum as seen by the pile at a specific depth. Its value therefore depends not only on the soil properties but also on the pile properties and geometry. Thus, the evaluation of the spring constant for that specific pile and soil type is complex and a large amount of engineering judgement is needed. It is worth noting that, in the evaluation of the kinematic response, the value of the stiffness coefficient has a relatively small influence on the maximum pile moment (the stiffness contrast between layers has a more significant role), and this may explain the relative success of the method. However, in evaluating the response of head-loaded piles (inertial effects), the stiffness coefficient plays a dominant role and hence the difficulties in selecting an appropriate value become apparent.

(b) *Group effects*
   The Winkler approach treats the soil as a series of independent springs (i.e. the displacement of one spring has no effect on the displacement of any other springs). This neglects continuity through the soil and makes it impossible to find a rational way to quantify the interaction effects between piles in a group. Thus, in evaluating inertial group effects, recourse is usually made to an extension of Poulos' static superposition approach to the dynamic case. However, the superposition of two-pile interaction factors is an approximate procedure which produces several limitations (e.g. Basile, 2003); its use becomes even more questionable in the dynamic environment where little calibration work has been carried out. With regard to kinematic group effects, these are usually ignored by Winkler models, despite the fact that some researchers have shown that such effects are not insignificant (Nikolaou *et al*., 2001; Dezi *et al*., 2009b, Elahi *et al*., 2010).

(c) *Load-deformation coupling*
   Pile-soil interaction is a three-dimensional problem and each of the load components has deformation-coupling effects. For example, a lateral load acting on a group of piles will also generate axial loads (as well as lateral loads) on the piles to counteract rocking of the pile group. This aspect, which is particularly important in real design (where the pile group is subjected to a combination of axial and lateral forces), cannot readily be modelled by the Winkler approach.

2.2 Continuum-based approach

The fundamental limitations of the Winkler model may be removed by means of rigorous continuum-based 3D dynamic solutions, generally based on the boundary element or the finite element method (e.g. Maheshwari *et al*., 2004). These analyses provide an efficient means of retaining the essential aspects of pile interaction through the soil continuum and hence a more realistic representation of the problem. However, these solutions are very complex to use for design purposes, particularly when non-linear behaviour is to be considered. Major problems are related to the high mesh dependency and to the uncertain in assigning mechanical properties to the pile-soil interface elements. In addition, such analyses are limited by the high computational costs which may be justified only for research work or for very large projects. This is particularly relevant for the dynamic case in which the disturbance travels as a wave in the ground and, contrary to the static case (where the load influence is confined to a limited area around the load application point), a very large area is affected. Thus, a FEM mesh generally needs to be very large to accommodate radiation damping and very dense to allow correct representation of prominent frequencies in the ground motion.

In order to overcome the shortcomings of dynamic analyses, practical pseudostatic approaches have recently emerged (e.g. Tabesh and Poulos, 2001; Liyanapathirana and Poulos, 2005; Elahi *et al*., 2010). These are based on a two-step analysis: computation of the soil movements via a free-field seismic analysis and then, by means of a static boundary element analysis, computation of the pile response subjected to the computed free-field soil movements (kinematic loading), in addition to the

static loading at the pile head (inertial loading). The rationale for the approach follows from the work of Tabesh & Poulos (2001) who demonstrated that, when the pile response is governed by the free-field ground movements (this is the case for relatively small cap-mass), the static interaction between pile and soil plays a dominant role, and an excellent agreement between the pseudostatic and dynamic analyses is observed. With the increase in cap-mass, the agreement between the pseudostatic and dynamic analyses is in some cases reduced, with a tendency of the pseudostatic approach to overestimate the pile internal forces by up to 25% (which is an acceptable conservatism for practical purposes). One reason is that, in the pseudostatic analysis, the maximum free-field effects and the maximum inertial effects have been assumed to act simultaneously (i.e. in phase), which does not occur in a dynamic analysis.

3. PGROUPN ANALYSIS

The pseudostatic approach recently proposed by Basile (2010) follows the procedure of Tabesh & Poulos (2001) for single piles in linear elastic soil, which has been extended to include the effects of group interaction and soil nonlinearity (via a hyperbolic continuum-based soil model). The numerical procedure is carried out within PGROUPN (Basile, 2003), a completely general computer program for determining the axial, lateral, rocking, and torsional response of pile groups by means of a boundary element formulation. The work makes use of the traditional Mindlin solution to perform a '*complete*' analysis of the group (i.e. the simultaneous influence of all the elements of all the piles within the group is considered), thereby removing the approximations of the interaction factor approach employed by Winkler models. The program has negligible computational costs and is widely used in pile group design through the commercial software Repute (Bond and Basile, 2009). The approach involves two main steps:

(1) Free-field site response analysis in order to obtain the maximum ground displacement profile along the pile and the maximum ground surface acceleration generated by the earthquake. For this purpose, the well-known SHAKE program or similar codes such as the EERA program (Bardet *et al.*, 2000) used herein, may be employed.

(2) Static analysis of the pile group, subjected to the computed maximum free-field ground displacement profile along the pile (kinematic loading) and to the pile cap load given by the cap-mass (representing the mass of the superstructure) multiplied by the maximum free-field ground surface acceleration (inertial loading). The analysis is based on a complete non-linear BEM formulation and involves discretization of the pile-soil interface into a number of elements, each element being acted upon by an unknown uniform stress (see Fig. 1). The method employs a substructure technique in which the piles and the surrounding soil are considered separately and then compatibility, equilibrium, and yielding conditions at the interface are imposed.
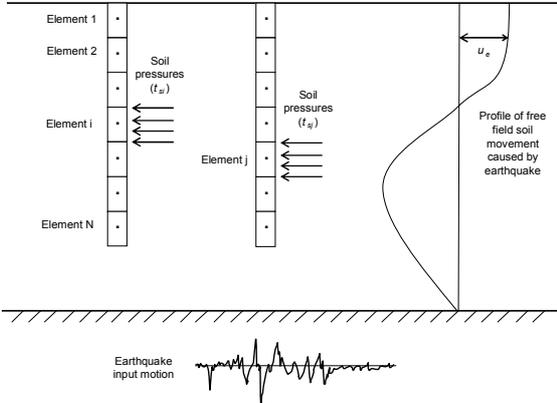
Figure 1. Pseudostatic BEM schematisation of the problem

4. NUMERICAL RESULTS

Numerical predictions from PGROUPN are compared with published results from alternative analyses. Attention will be focused on the internal load distribution along the piles due to kinematic interaction, in particular bending moments, whose evaluation may be critical in design.

4.1 Comparison with Nikolaou *et al*. (1995)

The beam-on-dynamic-Winkler-foundation (BDWF) approach proposed by Gazetas and his co-workers is one of the most established solutions for evaluating kinematic bending moments in seismically stressed piles (Nikoloau *et al.*, 1995, 2001; Mylonakis *et al.*, 1997; Gazetas and Mylonakis, 1998). Under the assumption of linear elastic soil behaviour, Nikolaou *et al.* (1995, 2001) carried out a comprehensive numerical study using three idealised soil profiles of actual sites (Fig. 2): (I) a 'Soft Clay' profile, i.e. a two-layer idealisation of an actual soil deposit consisting of 9.5 m of very soft clay underlain by stiff sand with a thickness of 20.5 m (ground type D according to EC8), (II) a 'Boston' profile, i.e. an idealisation of an actual profile from downtown Boston (ground type D), and (III) a 'Bay Area' profile, i.e. an idealised profile typical of the stiffer San Francisco Bay Area formations (ground type C). The pile lengths and shear wave velocity profiles are shown in Fig. 2, while a variation in damping ratio from 10% at the ground surface to 7% at the bottom was considered in all profiles. The piles are fixed-head with a diameter of 1.3 m, a mass density of 2.5 Mg/m^3 and a Young's modulus of 25 GPa. The acceleration time histories cover a broad range of possible rock motions and are taken from the PEER database. The input motions have been scaled to a peak bedrock acceleration of 0.10g and applied to the top of the bedrock.
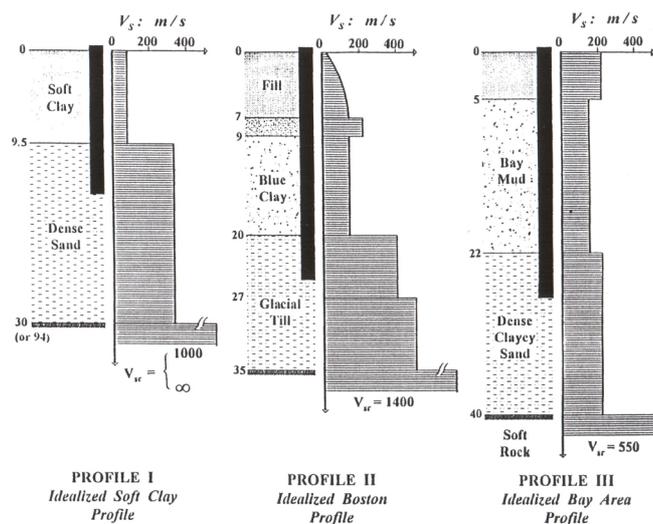
Figure 2. The three idealised soil profiles (after Nikolaou *et al*., 1995)

The kinematic bending moment distributions are shown in Fig. 3 in which, in order to facilitate the comparison with the pseudostatic PGROUPN analysis (continuous line), the envelope of the positive moments from the dynamic BDWF analysis by Nikoloau *et al*. (1995) (dotted lines) has been mirrored with respect to the pile axis. A number of trends emerges from this figure: (1) if kinematic effects are ignored, and only inertial loading at the pile-head is considered, this results in a considerable underestimation of bending moment at the pile-head. In addition, the effect of kinematic loading leads to significant moments at the interface between layers with different stiffness. This shows the crucial importance of accounting for both inertial and kinematic effects. (2) The above results suggest that the evaluation of kinematic bending moments may be important not only for ground types D or worse, as recommended by EC8, but also for ground type C. (3) The average differences between Nikolaou and PGROUPN predictions of maximum bending moment along the pile for the soil profiles (I), (II), and (III) are equal to +17%, -18%, and +16%, respectively, which is acceptable for practical pile design purposes. (4) The static PGROUPN profile in some parts matches the dynamic positive envelope and,

in other parts, the dynamic negative envelope. The fair agreement between the static and dynamic analyses suggests that the maximum values of soil displacement along the pile have occurred at the same time step in the free-field analysis (clearly, the static analysis is "blind" as to the direction of the developed moment because the absolute value of the maximum free-field displacements is used).
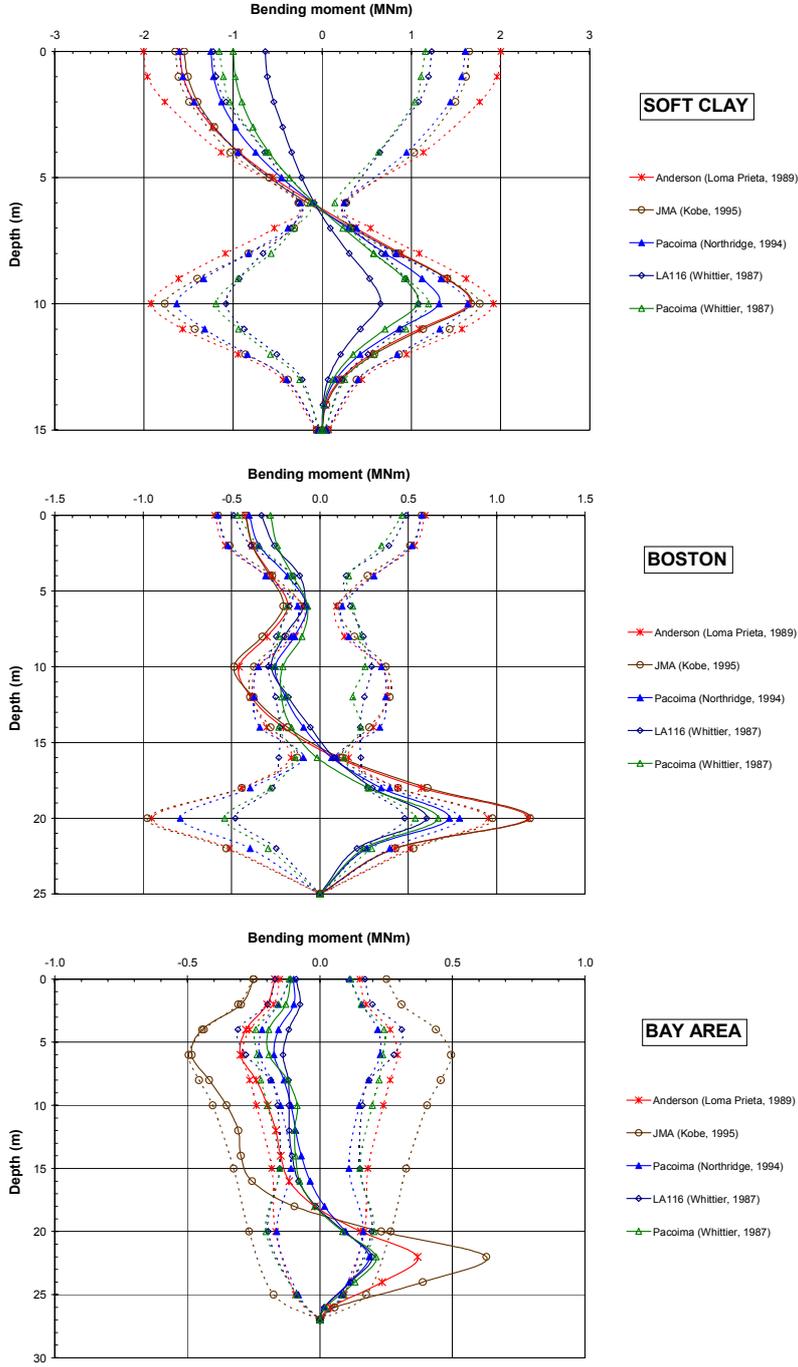
Figure 3. Bending moments from PGROUPN (continuous lines) and Nikolaou *et al.* (1995) (dotted lines)

The kinematic effects of pile-to-pile interaction as computed by PGROUPN are illustrated in Fig. 4 for 3x3 and 4x4 pile groups with a centre-to-centre spacing of three pile diameters. The analyses are carried out for the 'Soft Clay' profile, while the seismic motion is JMA (Kobe, 1995). For comparison, the shear force and bending moment profiles computed for the single isolated pile have been included (no axial force is induced on the single pile due to horizontal ground movement). The following characteristics of behaviour can be discerned: (1) effects of kinematic group interaction lead to a decrease of bending moment at the pile head and at the layer interface as compared to the single pile;

(2) the maximum bending moment is found at the head of the central piles in the pile groups, whereas it occurs at the layer interface in the single pile; (3) the reduction in maximum bending moment from the single pile to the pile group is equal to 22% for the 9-pile group and to 27% for the 16-pile group; (4) the corner piles of the group carry the greatest proportion of shear force with a reduction in maximum values from the single pile to the pile group equal to 29% for the 9-pile group and to 35% for the 16-pile group; (6) although no vertical load is applied to the group, axial forces develop in the piles to counteract rocking of the pile group; (7) increasing the number of piles in the group (i.e. from 9 to 16) leads to a reduction of the maximum values of axial force, shear force and bending moment. Overall, the results show the importance of considering the effects of kinematic pile-to-pile interaction in order to obtain a more realistic (and often more economical) prediction of pile group behaviour. This is in contrast with common practice of ignoring kinematic group effects.
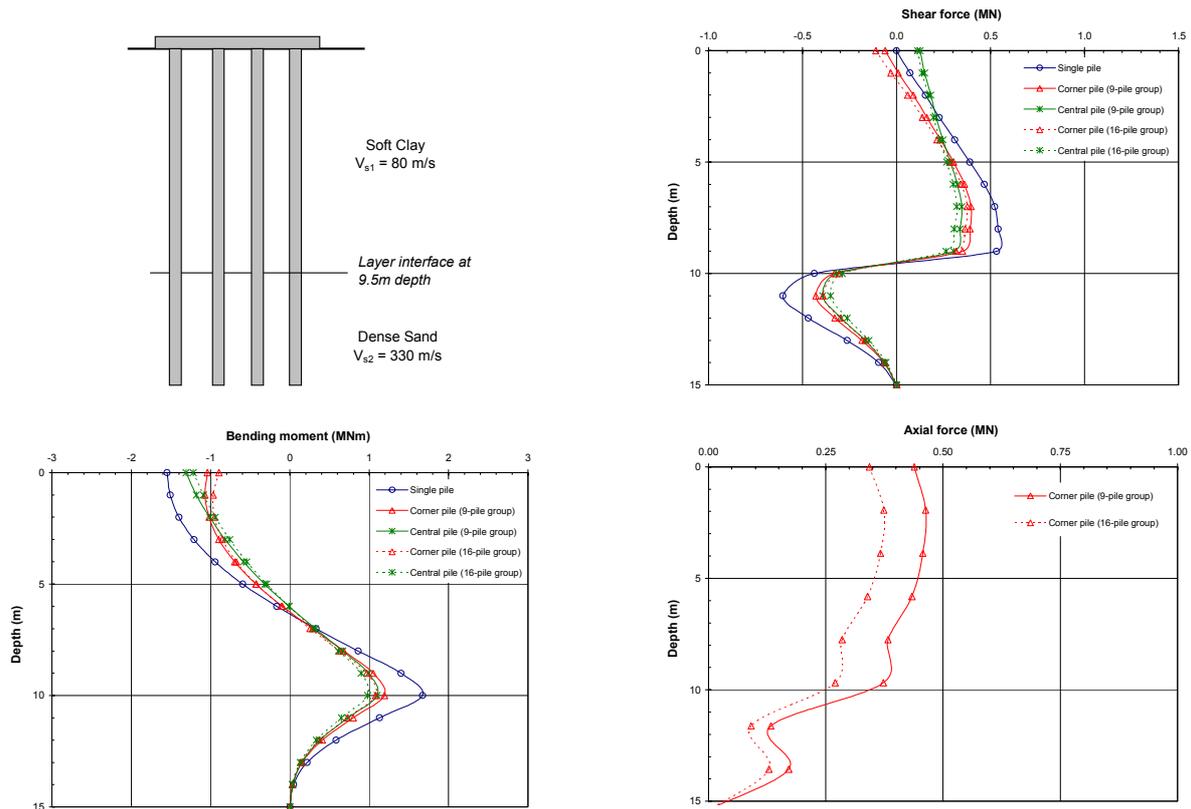
Figure 4. PGROUPN prediction of internal forces in 3x3 and 4x4 pile groups

Finally, Fig. 5 illustrates the effect of soil nonlinearity on the kinematic bending moment induced on the single isolated pile and on the corner pile of the 3x3 group. The results have been computed by PGROUPN under the assumption that the clay material of the 'Soft Clay' profile has an undrained shear strength (C_u) of 30 kPa, a buoyant unit weight of 9 kN/m^3 and an adhesion factor (α) of 0.6 (the latter parameter is required to evaluate the non-linear rocking response of the group), while the stiff sand is characterised by a friction angle of 40 degrees, a pile-soil interface angle (δ) of 32 degrees, a buoyant unit weight of 10 kN/m^3 and a coefficient of horizontal soil stress (K_s) of 1.0. The PGROUPN non-linear analysis has been preceded by a non-linear free-field EERA analysis using an initial damping ratio D_o equal to 0.5% and the default degradation curves for the shear modulus and damping ratio of sand and clay. Fig. 6 shows the maximum acceleration and lateral movement profiles obtained from the linear elastic (LE) and non-linear (NL) free-field EERA analyses. For comparison, the moment profiles from the linear elastic PGROUPN analyses (based on the linear elastic EERA analysis), as already reported in Fig. 4, are also included in Fig. 5. In addition, the results from a linear PGROUPN analysis based on a non-linear EERA analysis are shown. A number of features emerges from this figure: (1) kinematic pile-to-pile interaction reduces leads to a reduction of 27% of the maximum bending moment as compared to a single isolated pile, thereby confirming the trend

observed in the case of linear elastic soil; (2) consideration of soil nonlinearity has a significant influence on the kinematic moment of both single piles and pile groups, leading to an increase of maximum moment equal to 42% for the single pile and to 43% for the corner pile; (3) the difference between the single-pile moment profile obtained from the non-linear PGROUPN analysis (preceded by a non-linear EERA analysis) and that obtained from the linear elastic PGROUPN analysis (preceded by an identical non-linear EERA analysis) shows that the overall nonlinearity of response is determined not only by the nonlinearities due to the shear waves propagating in the free-field soil but also by the nonlinearities due to pile-soil interaction.
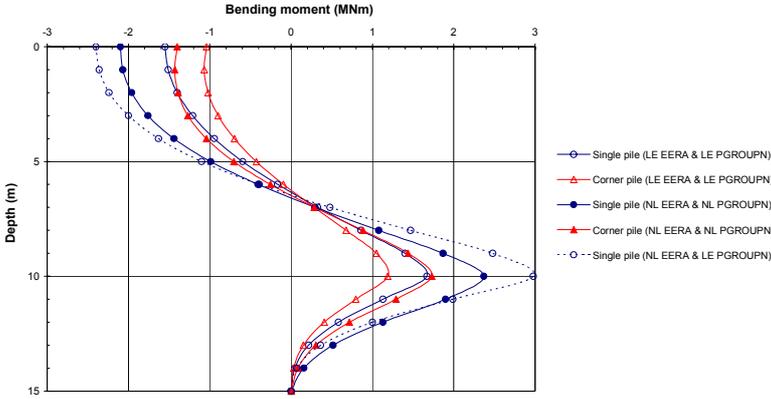
Figure 5. Influence of soil nonlinearity on bending moment distribution
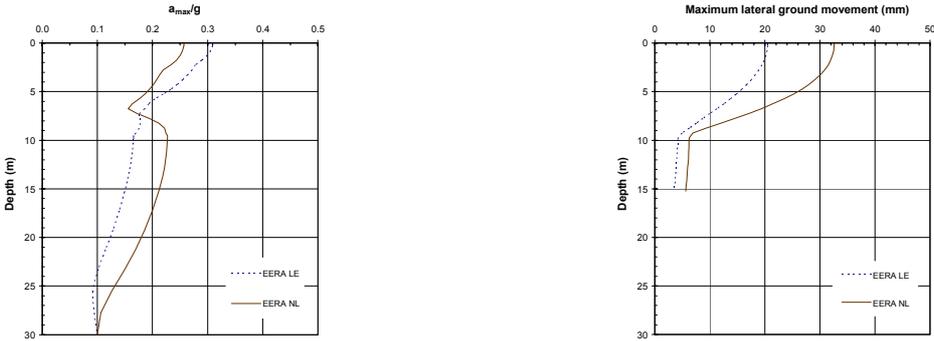
Figure 6. Maximum free-field acceleration and lateral movement profiles

4.2 Comparison with Dezi *et al*. (2009)

A BDWF solution for the kinematic interaction analysis of pile foundations has been proposed by Dezi *et al*. (2009b) in which a finite element model is used for the piles and a linear elastic Winkler-type approach is considered for the soil. The kinematic bending moment profiles reported by Dezi *et al*. (2009a) are compared with those predicted by PGROUPN for two single piles with a variable diameter (D = 600, 800 mm) and embedded into two different soil profiles (see Figs. 7-8). The piles are fixed-head with a length of 24 m, a mass density of 2.5 Mg/m^3 and a Young's modulus of 30 GPa. The seismic action is defined with reference to outcropping bedrock by considering an artificial accelerogram matching the EC8 Type 1 elastic response spectrum for ground type A (EN 1998-1). The maximum free-field lateral movement profiles of Figs. 7-8 (as reported by Dezi and colleagues) have directly been used as an input to the PGROUPN pseudostatic analysis. The kinematic bending moment profiles for the two-layer and the three-layer soil profiles are compared in Figs. 7 and 8, respectively. The figures also include the moment profiles obtained by Dezi and colleagues using a pseudostatic approach based on a Winkler-type model. A good agreement between PGROUPN and the BDWF approach by Dezi and colleagues is observed, with differences in maximum bending moments ranging from 2% (two-layer profile with D = 800 mm) to 18% (three-layer profile with D = 800 mm). It is worth noting that the '*Winkler*' pseudostatic approach compares very closely with the '*continuum-based*' pseudostatic approach of PGROUPN.
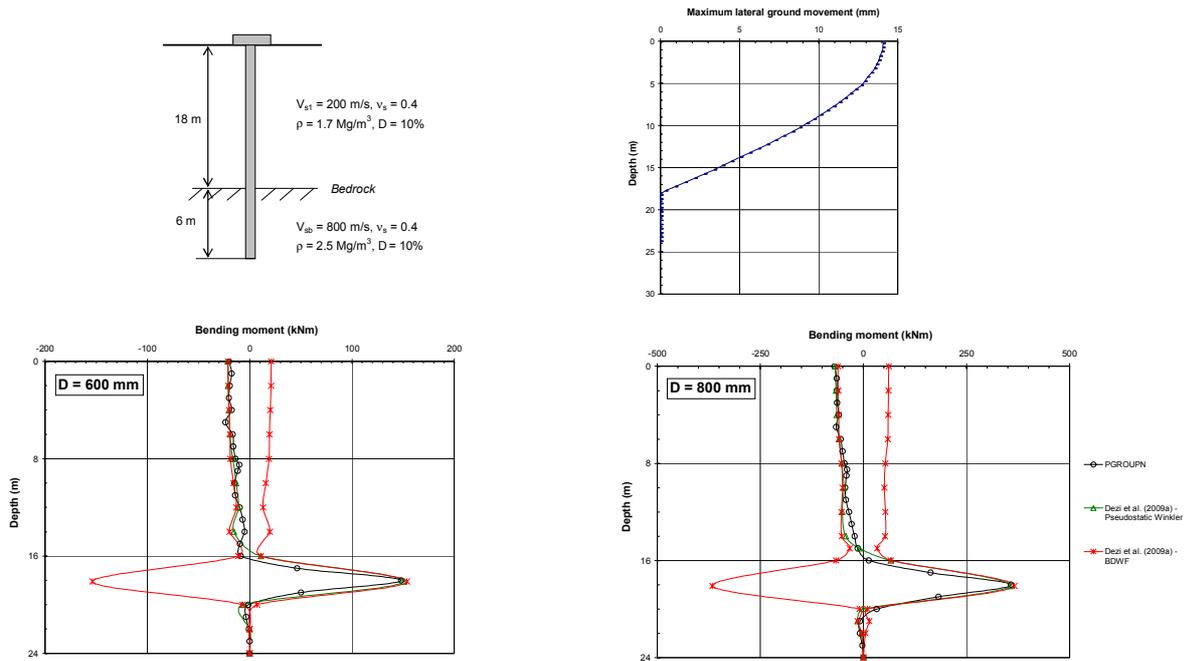
Figure 7. Maximum free-field ground movement and bending moment profiles for two-layer soil
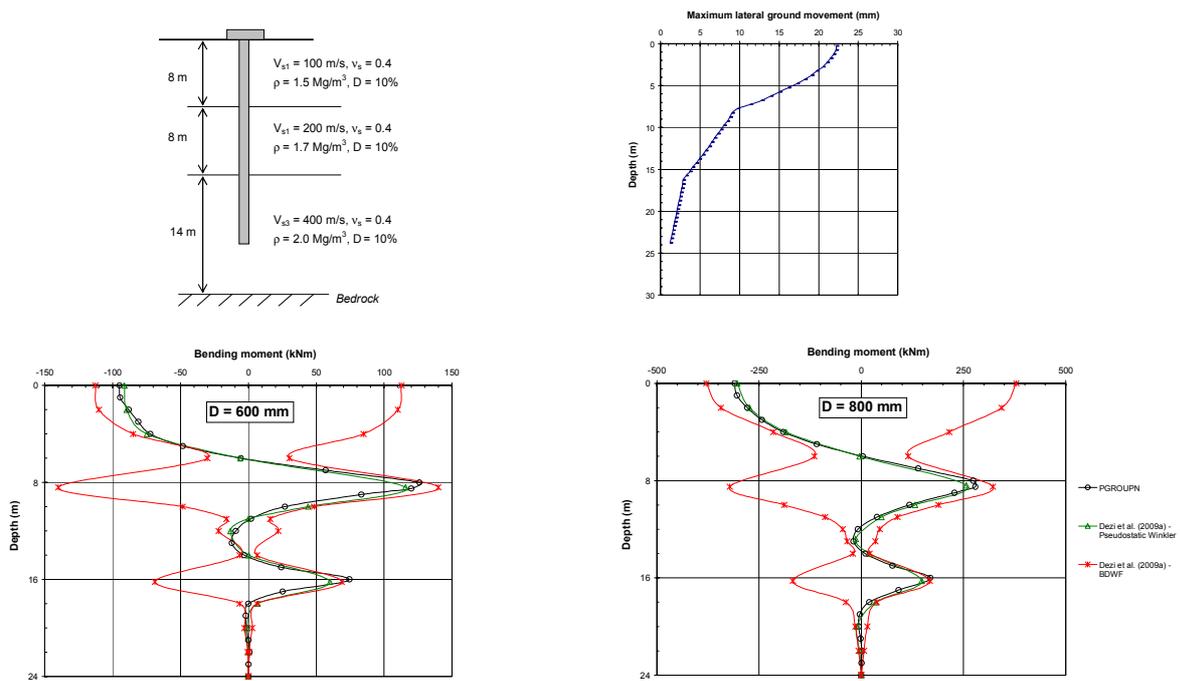
Figure 8. Maximum free-field ground movement and bending moment profiles for three-layer soil

5. CONCLUDING REMARKS

The effects of kinematic interaction on the pile internal forces have been examined. Based on the results presented in the paper, a number of considerations may be made:

• The PGROUPN pseudostatic approach recently proposed by the author (Basile, 2010) compares favourably with dynamic Winkler-type (BDWF) analyses. The maximum average differences in kinematic bending moments predicted by the two approaches are within 18%, which is acceptable for practical pile design purposes.

● Kinematic effects can have a significant influence on the bending moment at the pile head and at the interface between soil layers with marked stiffness contrast. Such effects may be important not only for ground types D or worse, as recommended by Eurocode 8, but also for ground type C.

● Kinematic pile-to-pile interaction, generally ignored by current analysis methods, has an effect on the pile-group load distribution, specifically by decreasing the induced shear forces and bending moments (i.e. a beneficial effect), and by increasing the induced axial forces (i.e. a detrimental effect), as compared to a single isolated pile.

● In addition to the nonlinearity arising from the passage of the seismic waves in the free-field soil, nonlinearity due to pile-soil interaction can have a significant influence on the distribution of kinematic pile internal forces and should not routinely be disregarded.

● Pile-soil interaction is a three-dimensional problem and each of the load components has deformation-coupling effects. Modelling of this aspect is crucial in real design (where the pile group is normally subjected to a combination of axial forces, lateral forces and bending moments, in addition to inertial and kinematic loading), thereby allowing a more realistic prediction of pile-group response.

● The results confirm that the PGROUPN pseudostatic analysis has promise in practical applications, offering a reasonable compromise between the fundamental limitations of Winkler models and the complexity and time-consuming nature of rigorous dynamic FEM or BEM analyses.

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