TORSIONAL RESPONSE OF PILE GROUPS

Francesco Basile, Geomarc Ltd, Messina, Italy

Tall buildings play a key role in current urban strategies and regeneration. Development of these buildings presents several geotechnical problems related to the design and assessment of pile foundations. Among these, the transmission of torsional forces to the piles due to the eccentricity of wind action is of particular interest. In this paper, a numerical method of analysis is introduced for the determination of the non-linear response of single piles and pile groups to torsional loading. Application of the proposed method, as implemented in the computer program PGROUPN, is illustrated through comparison with alternative numerical analyses, analytical solutions, centrifuge model tests and published case histories.

INTRODUCTION

Pile foundations of some structures, such as tall buildings, bridge piers, offshore platforms and electric transmission towers, can be subjected to significant torsional forces due to eccentric lateral loading from ship impacts, high-speed vehicles, wind and wave actions, and other sources of loading. Inadequate design of the piles against torsional loads may seriously affect the serviceability and safety of these structures with catastrophic consequences. The literature reports two cases of tall buildings in Miami and in Lubbock (Texas) which have suffered serious damage due to wind action and exhibited marked permanent deformations from torsion (Vickery, 1979). Another case, described by Barker & Puckett (1997), reports the collapse of a support pier of the 6.82km long Sunshine Skyway Bridge in Florida caused by the eccentric impact of a bulk carrier. About 395m of the bridge fell into the sea, resulting in thirty-five deaths. It is therefore important that the strength and deformation characteristics of the foundation piles are properly addressed in design in order to ensure safety and cost-effectiveness.

When a pile group is subjected to torsion, its response is mainly governed by the interaction between the torsional and lateral behaviour of the individual piles (*see* Fig. 1). Thus, the total applied torque on the group (T) will be shared by the pile torsional component (T_i) plus the lateral contribution from the pile shear force H_iS_i (where H_i is the pile shear force and S_i is the distance of the pile from the torsional centre of the group). Several investigators have carried out model testing and developed numerical solutions for the analysis of pile foundations under torsion. However, these studies are mainly concentrated on the torsional response of single piles while little attention has been paid to the effects of group interaction between piles.

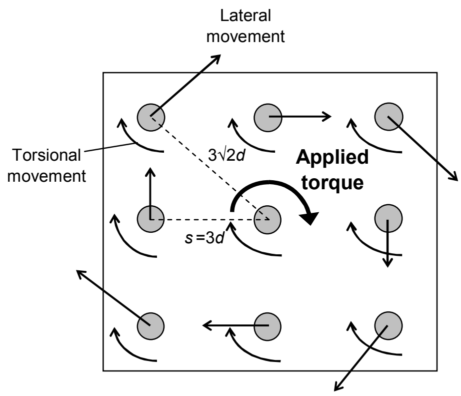

Figure 1: Movements in 3x3 group under torque

Poulos (1975) presented a continuum-based solution, using the boundary element method, for the elastic response of single piles under torque. Using a load-transfer approach, Randolph (1981) derived closed-form solutions for the torsional stiffness of a single pile based on a simple assumption concerning the stress field around a pile undergoing torsion. The analysis was then extended by Guo & Randolph (1996) in order to include a more general non-homogeneity of the soil profile and the non-linear soil response using a hyperbolic stress-strain law. Chow (1985) presented a discrete element approach in which the pile is modelled as a series of elements and the soil is treated as a series of independent layers, each with a modulus of subgrade reaction.

As for experimental studies, several investigators carried out conventional 1-g model tests of single piles subjected to torsion. Poulos (1975), as well as Georgiadis & Saflekou (1990), carried out model tests of single piles in clay to investigate the relationship between axial and torsional response of a pile. The results of the tests indicated that there was a good correlation between the pile-soil adhesion from each type of test and that the soil shear modulus calculated from axial load tests could reasonably be used to predict the response of piles under torsion, at least at ordinary working load levels. Another interesting insight reported by Poulos (1975) is that no significant interaction effect between the torsionally loaded model piles was observed, even at a pile spacing of about three pile diameters.

Recently, Kong (2006), Zhang & Kong (2006) and Kong & Zhang (2007, 2008) reported a series of centrifuge model tests on torsionally loaded single piles and pile groups in sand. From these studies, it was found that a pile group subjected to torsion simultaneously mobilizes lateral and torsional resistance on the individual piles (*see* Fig. 1), and the torsional contribution to the total applied torque is in a range of 20-50%. The tests also showed no evidence of significant interaction with respect to the effect of torsional movement on the torsional behaviour of adjacent piles located at a distance of three pile diameters, thereby confirming the previous finding by Poulos (1975). In addition, Kong (2006) and Kong & Zhang (2009) proposed an empirical method to analyse the response of pile groups to torsion in which load-transfer curves are used for single pile response, while Mindlin (1936)'s and Randolph (1981)'s solutions are adopted to evaluate group interaction.

No full-scale tests on torsionally loaded pile groups have been reported in literature. The only available results from field tests refer to single piles and have been described by Stoll (1972), who devised a simple device for testing cylindrical piles and conducted tests on two steel pipe piles.

Computer programs for pile-group analysis

For pile groups, the complexity of the problem depicted in Fig. 1, which is fully three-dimensional, generally requires the use of computer-based methods of analysis. To date, several computer programs are available in order to estimate the deformations and load distributions among the individual piles of a group subjected to general loading conditions, including torsion.

As described by Basile (2003), these programs may be broadly classified into two categories:

(1) continuum-based approaches;
(2) load-transfer (or subgrade reaction) approaches.

The latter category, including programs such as GROUP (Reese et al., 2000), is based on the so-called Winkler idealisation of the soil (i.e. the soil is modelled as a series of independent springs). This method is attractive in its flexibility, enabling non-linear and inhomogeneous soil conditions to be incorporated easily. In the case of torsional loading, the program is based on "τ-θ" curves, where τ is the torsional shear stress and θ is the local twist angle of the pile shaft. The approach is similar to that adopted for the axial and lateral response, based on "*t-z*" and "*p-y*" curves, respectively. However, by disregarding soil continuity, such an approach oversimplifies the problem and makes it impossible to find a rational way to quantify the interaction effects between piles in a group. Thus, in evaluating group effects, recourse is made to an entirely empirical procedure in which the single pile load-transfer curves are modified on the basis of empirical parameters (e.g. the "*p*-multipliers" for lateral loading) derived from a limited number of small-scale and full-scale experiments. Thus, many uncertainties remain on the general use of the approach in routine design (Rollins et al, 1998, 2000; Basile, 2003; Finn, 2005). Another limitation is the selection of soil parameters in that the key input parameter (i.e. the modulus of subgrade reaction, k) is not a fundamental soil property but is also dependent on the dimensions of the pile. Thus, the modulus of subgrade reaction is an empirical parameter which can only be determined with sufficient confidence by back-figuring from the results of a field test on an instrumented pile. In conclusion, the load-transfer approach may be regarded as a link between the interpretation of full-scale pile tests and the design of similar single piles rather than as a general tool for pile group design.

A more rational approach is offered by soil continuum-based solutions which make use of finite element (FEM), finite difference (FDM) or boundary element (BEM) methods. These solutions provide an efficient means of retaining the essential aspects of pile interaction through the soil continuum and hence a more realistic representation of the problem. Further, the mechanical characteristics to be introduced into the model have now a clear physical meaning (e.g. the soil Young's modulus, E_s) and they can be measured directly in soils investigation.

Finite element and finite difference solutions are the most powerful numerical tools for the analysis of pile groups. However, when applied to routine design problems, these methods suffer from significant limitations, mainly related to the high computational costs and their complexity (e.g. high mesh dependency and uncertain in assigning stiffness/strength properties to the pile-soil interfaces), particularly if non-linear soil behaviour is to be considered. By contrast, BEM provides a complete problem solution in terms of boundary values only, specifically at the pile-soil interface. This leads to a drastic reduction in unknowns to be solved for, thereby resulting in substantial savings in computing time and data preparation effort. This feature is particularly important for three-dimensional problems such as pile groups.

Among the pile-group programs based on the boundary element method, PIGLET (Randolph, 1980), CLAP (an extension of DEFPIG by Poulos, 1990) and GEPAN (Xu & Poulos, 2000) are able to deal with torsional loading, as well as other types of loading. PIGLET and CLAP adopt the simplified solution by Randolph (1981) for the single-pile response to torsion, while group effects due to axial and lateral loading are modelled using the interaction-factor approach (i.e. by calculating the influence coefficients for each pair of piles and by merely superimposing the effects). However, this approach produces a number of limitations, including approximations in the evaluation of load and moment distributions along the piles, and an overestimate of interaction effects between piles (as the stiffening effect of intervening piles in a group is neglected). The effects of torsional interaction between piles in a group are not considered. A more rigorous BEM analysis, which takes into account the simultaneous presence of all elements of all piles within the group (i.e. the so-called "complete" analysis), is performed by the numerical code GEPAN. In this approach, the cylindrical elements at the pile-soil interface are in turn divided into partly cylindrical or annular sub-elements. The program is restricted to the linear elastic range.

In this paper, a new method of analysis for single piles and pile groups subjected to torsional loading is introduced. The proposed method is an extension of the boundary element formulation described by Basile (2003) for axially and laterally loaded pile groups, and implemented in the computer program PGROUPN (the calculation engine of the pile-design software Repute by Geocentrix, 2009). The main feature of PGROUPN lies in its capability to provide a "complete" non-linear BEM solution of the soil continuum (while retaining a computationally efficient code), thereby overcoming the approximations which occur with the traditional interaction factor and load-transfer approaches. It should be emphasised that use of a non-linear soil model is of basic importance in pile-group design as it avoids exaggeration of stresses at pile group corners (a common limitation of linear elastic models), and therefore reduces consequent high loads and moments, even at typical working load levels. Benefits of this include an improved understanding of pile group behaviour and thus more effective design techniques.

METHOD OF ANALYSIS

The proposed approach, based on the complete boundary element method, employs a sub-structuring technique in which the piles and the surrounding soil are considered separately and then compatibility and equilibrium conditions at the interface are imposed. The method involves discretisation of the pile-soil interface into a number of cylindrical elements, each element being acted upon by an unknown uniform stress, as shown in Fig. 2a. If the pile base is assumed to be smooth, the effects of the torsional components of stresses over the base area can be ignored. Such an assumption is in line with previous findings by Poulos (1975) and Randolph (1981) who found that very little torque is transferred to the pile base (even for a short rigid pier).

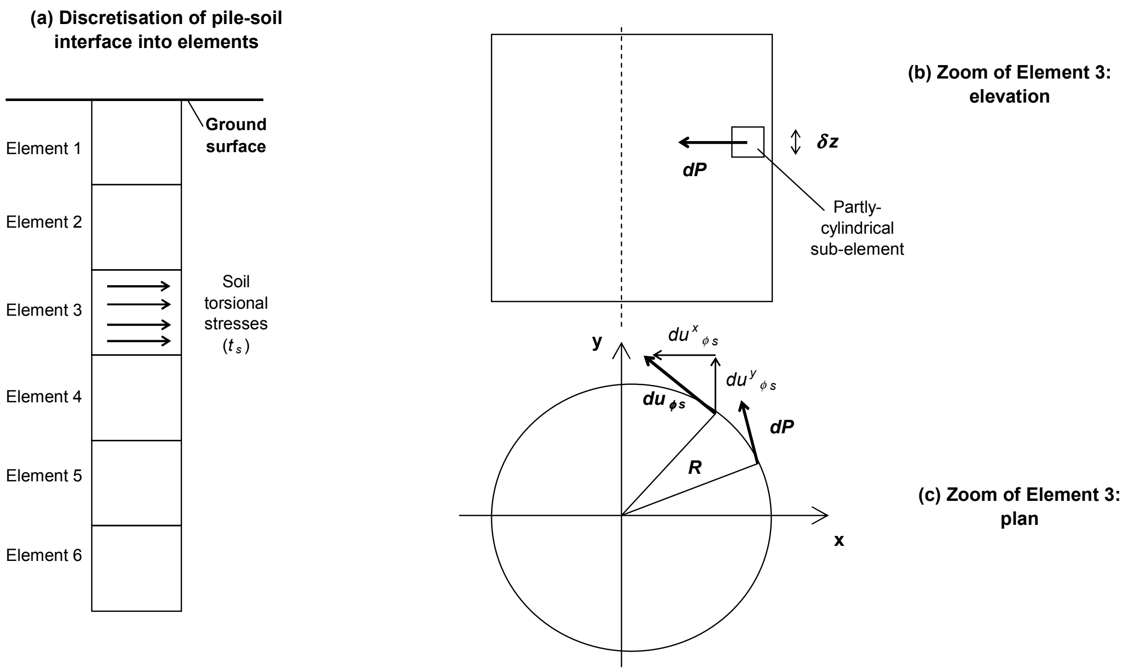

Figure 2: Definition of problem

Soil domain

The boundary element method involves the integration of an appropriate elementary singular solution for the soil medium over the surface of the problem domain, i.e. the pile-soil interface. With reference to the present problem, the well-established solution of Mindlin (1936) for a point load within a homogeneous, isotropic elastic half space has been adopted to correlate the soil rotations and the corresponding soil torsional stresses:

$$\{\phi_s\} = [G_s]\{t_S\} \qquad \text{(Equ. 1)}$$

where ϕ_s are the soil rotations, t_s are the soil torsional stresses and G_s is the soil flexibility matrix obtained from the integration of Mindlin's solution over a cylindrical surface.

Considering a small, partly-cylindrical sub-element at the pile-soil interface (*see* Figs. 2b and 2c), the soil rotation $d\phi_s$ can be calculated from the soil deflection du_{ϕ_s} tangential to the pile surface as $d\phi_s = du_{\phi_s}/R$ (where R is the pile radius). The tangential deflection du_{ϕ_s} is then expressed as the sum of its components in the x and y directions (Poulos, 1975), i.e.:

$$du_{\phi_s} = du_{\phi_s}^x + du_{\phi_s}^y \qquad \text{(Equ. 2)}$$

where $du_{\phi_s}^x$ and $du_{\phi_s}^y$ are the components of deflection in the x and y directions, respectively, and are evaluated from the equations of Mindlin for a horizontal subsurface point load dP. By integration of Equ. (2) over a cylindrical element, the tangential deflection, and thus rotation, for all the elements at the pile-soil interface are obtained. It should be observed that Mindlin's solution is strictly applicable to homogeneous soil conditions. However, non-homogeneous soil profiles may be treated using the classical averaging procedure proposed by Poulos (1979) and widely adopted in practice (i.e. in the evaluation of the influence of one loaded element on another, the value of soil modulus is taken as the mean of the values at the two elements). This procedure is adequate in most practical cases.

Pile domain

If the piles are assumed to act as simple beam-columns which are fixed at their heads to the pile cap, the rotations and torsional stresses over each element can be related to each other via the elementary beam theory, yielding:

$$\{\phi_p\} = [G_p]\{t_p\} + \{B\} \qquad \text{(Equ. 3)}$$

where ϕ_p are the pile rotations, t_p are the pile torsional stresses, B are the pile rotations due to unit boundary displacements and rotations of the pile cap, and G_p is a matrix of coefficients obtained from the classical beam theory for a cylinder.

Solution of the system

The soil and pile equations (1) and (3) may be coupled via compatibility and equilibrium constraints at the pile-soil interface. Thus, by specifying unit boundary conditions, i.e. unit values of vertical displacement, horizontal displacement and rotation of the pile cap, these equations are solved, thereby leading to the distribution of stresses, loads, torques and moments in the piles for any loading condition.

Limiting pile-soil stresses

The foregoing procedure is based on the assumption that the soil behaviour is linear elastic. In practice, however, as the pile is twisted, the torsional stress on the pile-soil interface will reach a limiting value, t_{lim}. In the present analysis, this limiting value is assumed to be equal to the available pile-soil shaft friction. For cohesive soils, following a total stress approach, the limiting torsional stress may be expressed as:

$$t_{\lim} = \alpha C_u \qquad \text{(Equ. 4)}$$

where C_u is the undrained shear strength of the soil and α is the adhesion factor. For cohesionless soils, following an effective stress approach, the limiting torsional stress is taken as:

$$t_{\lim} = K_s \sigma_v' \tan\delta \qquad \text{(Equ. 5)}$$

where K_s is the coefficient of horizontal soil stress, σ_v' is the effective vertical stress and δ is the angle of friction between pile and soil.

Extension to non-linear soil behaviour

Non-linear response of the soil is modelled, in an approximate manner, by assuming that the soil Young's modulus varies with the stress level at the pile-soil interface. Similarly to the approach employed for the axial response, the hyperbolic stress-strain law introduced by Duncan & Chang (1970), and applied to the torsional case by Guo & Randolph (1996), has been adopted:

$$E_{\tan} = E_i\left(1 - \frac{R_f t}{t_{\lim}}\right)^2 \qquad \text{(Equ. 6)}$$

where E_{tan} is the tangent soil modulus, E_i is the initial tangent soil modulus, R_f is the hyperbolic curve-fitting constant, t is the pile-soil torsional stress and t_{lim} is the limiting value of pile-soil torsional stress defined in Equs. (4) and (5). Thus, the boundary element equations described above for the linear response are solved incrementally using the modified values of soil Young's modulus of Equ. (6) and enforcing the conditions of yield, equilibrium and compatibility at the pile-soil interface.

The hyperbolic curve fitting constant R_f defines the degree of curvature of the stress-strain response and its value can range between 0 (an elastic-perfectly plastic response) and 0.99 (R_f = 1 is representative of an asymptotic hyperbolic response in which the limiting pile-soil torsional stress is never reached). The most reliable method to determine the value of R_f is by back-fitting the PGROUPN torque-twist angle curve at the pile-head with the measured data from a full-scale torsional pile load test. In the absence of any test data, the values of R_f can be assessed based on experience and, from the results illustrated in this paper, a value of R_f = 0.99 appears to provide a reasonable estimate. It is worth noting that a similar high value (i.e. R_f = 0.95) is recommended by Guo & Randolph (1996) using a similar non-linear model, thereby confirming that, in the case of torsional loading, the effect of non-linearity close to the pile is much more localised than in the axial case (where values of R_f in the range 0-0.5 are generally appropriate).

COMPARISON WITH NUMERICAL AND EXPERIMENTAL RESULTS

Numerical results from the preceding PGROUPN analysis are validated through a comparison with alternative numerical analyses, analytical solutions and centrifuge model tests.

Comparison with Xu & Poulos (2000)

In order to investigate the twist angles (ϕ_T) at the head of a torsionally loaded single pile, Figure 3 compares PGROUPN results with those of the complete BEM analysis of GEPAN, as reported by Xu & Poulos (2000). The figure also shows the results obtained from PIGLET, which adopts the analytical solution by Randolph (1981). The pile has a diameter of 1 m, is embedded in a linear-elastic homogeneous soil (E_s = 20 MPa, where E_s is the soil Young's modulus) and is subjected to a torsional moment of 1 MNm. Results are expressed as a function of the pile slenderness ratio L/d (where L is the pile length and d its diameter) and the pile-soil relative stiffness K= E_p/E_s (where E_p is the pile Young's modulus). A very good agreement between the complete BEM solutions of GEPAN and PGROUPN is observed, while PIGLET gives higher values of the twist angles, particularly for rigid piles.

Turning to pile-group behaviour, Figure 4 shows the torque distribution, expressed as the ratio of pile-head torque T to pile-head average torque T_{av}, predicted by PGROUPN, GEPAN and PIGLET for a 3x3 pile group under torsional loading (with L/d = 25, K= E_p/E_s =1500 and soil Poisson's ratio of 0.5).

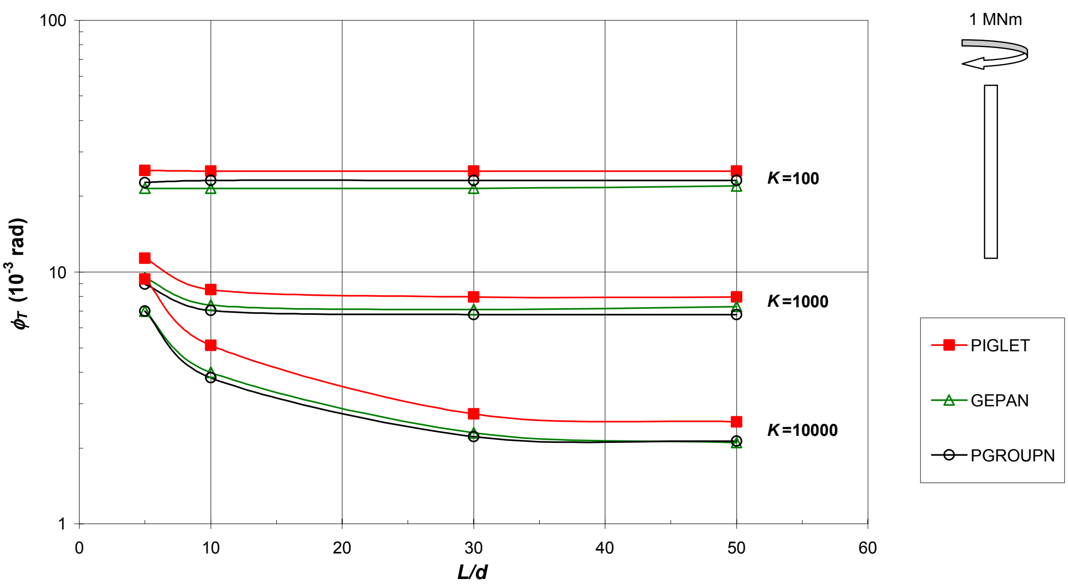

Figure 3: Comparison of the twist angle at head of torsionally loaded single pile

Results are expressed as a function of the normalised pile spacing (*s/d*), where *s* is the centre-to-centre pile spacing. Figure 4 also includes the predictions from a version of PGROUPN which ignores the effects of torsional interaction between the individual piles of the group (similarly to the approach followed by PIGLET), thereby resulting in a uniform torque distribution between piles. When such interaction effects are taken into account within the GEPAN and PGROUPN analyses, this leads to different torsional contributions among the piles, depending on the pile location within the group. Similarly to the case of axially and laterally loaded pile groups, the corner piles carry the greatest proportion of torque, the central pile carries the least, and the side piles are in between. Some discrepancies between the values predicted by PGROUPN and GEPAN are observed in Figure 4, owing to the different analysis methods employed by the two programs. It is found that the percentage of torsional contribution to resist the total applied torque is higher in PGROUPN than in GEPAN (in which the contribution of the lateral component will be higher).

Table 1 compares the main results obtained from PIGLET and PGROUPN for the 3x3 pile group of Figure 4, using a typical pile spacing *s/d* = 3 and applying a total torque of 900 kNm. While the values of twist angle and maximum torque are similar, some differences in the lateral load and bending moment predictions are observed. These may be explained with the different approaches adopted by the two programs: PIGLET, as well as ignoring torsional interaction effects, is based on superposition of two-pile interaction factors, whereas PGROUPN performs a complete analysis of the entire group, thereby considering in a more rigorous fashion the coupling between lateral and torsional loading (this feature of behaviour is usually referred to as the load-deformation coupling effect). It is worth noting that consideration of torsional group effects within the PGROUPN analysis leads to an increase of the twist angle, maximum lateral load and maximum bending moments of about 7%. Finally, it should be emphasised that the above PIGLET and GEPAN solutions are restricted to piles embedded in a linearly elastic soil. However, that is far from realistic for real soil, whose behaviour is highly non-linear, even at low load levels. The effects of soil nonlinearity on the torsional response of piles will be examined in the next sections.

Table 1: Comparison of solutions for 3x3 pile group under torque

Quantity	**PIGLET**	**PGROUPN** (no torsional group effects)	**PGROUPN**
Twist angle (radx10^{-4})	1.22	1.38	1.47
Maximum lateral load in x dir. (kN)	24.5	40.3	43.0
Maximum lateral load in y dir. (kN)	24.5	40.3	43.0
Maximum bending moment in xz plane (kNm)	48.9	79.8	85.0
Maximum bending moment in yz plane (kNm)	48.9	79.8	85.0
Maximum torque (kNm)	15.6	20.8	16.9

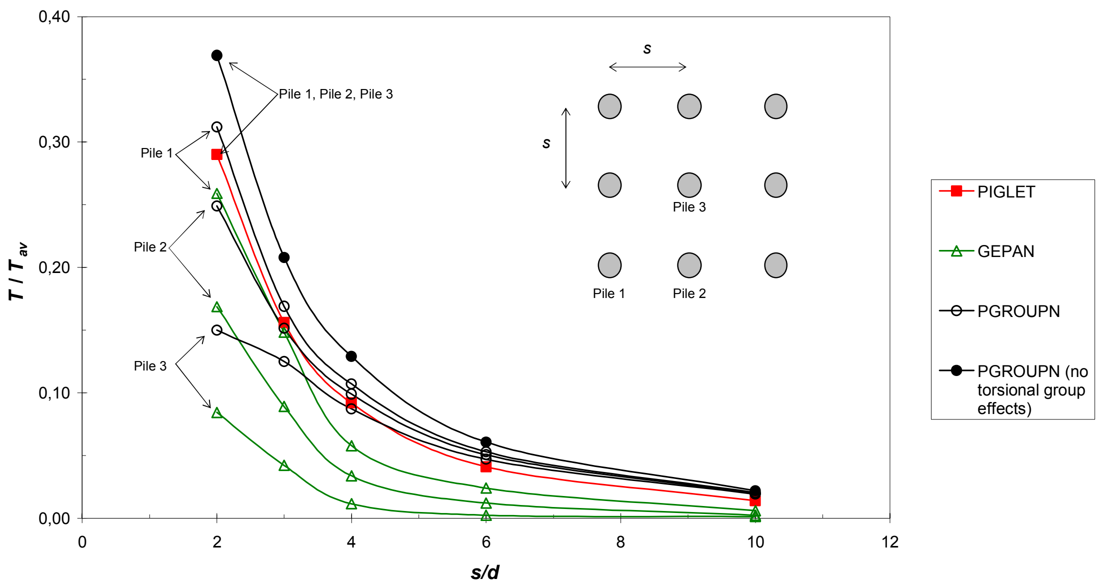

Figure 4: Load distribution of torsion loading in 3x3 pile group

Comparison with centrifuge tests of Kong & Zhang (2009)

Kong (2006), Zhang & Kong (2006) and Kong & Zhang (2007, 2008) reported a series of centrifuge model tests on torsionally loaded single piles and pile groups in loose and dense sand. Although some problems related to scale effects may still be present, centrifuge testing is an improved method of physical modelling over the conventional model tests in 1-g condition in that it is able to better reproduce the stress conditions that would exist in a full-scale installation. A summary of the main input parameters for the single pile and the pile groups is reported in Table 2 below. For convenience, input data and results are reported in prototype scale.

Figures 5 and 6 show a good agreement between test data and PGROUPN predictions for the torque-twist angle curves and torque distributions of the single piles. It is of interest to observe that the non-linear soil model adopted by PGROUPN is capable of capturing the main non-linear features of behaviour from the centrifuge tests. Figure 5 also includes the numerical predictions obtained from the empirical approach by Kong & Zhang (2009), based on back-analysis. In this approach, single-pile response is modelled by means of load-transfer curves (i.e. *p-y* curves for the lateral response and τ-θ curves for the torsional response), including an empirical factor (β) to model the interaction between the lateral and torsional response, while group effects are modelled using the linear elastic solutions of Mindlin (1936) and Randolph (1981). It should be emphasised that, in addition to the parameters shown in Table 2 (which follow those reported by Kong & Zhang), the soil parameters adopted for the PGROUPN analyses include an uniform soil modulus (E_s) of 7 MPa and a coefficient of horizontal soil stress (K_s) of 0.6 for the loose sand, with the corresponding values for dense sand being equal to E_s = 11 MPa and K_s = 1.1. The hyperbolic curve fitting constants (R_f) for the lateral and torsional response were both taken as 0.99, while the interface angle of friction between pile and soil (δ) is taken as 0.8ϕ for both loose and dense sands.

Table 2: Input parameters for comparisons of Figs. 5-10

	Pile embedded length, m	Pile free-standing length, m	Pile diameter, m		Pile torsional rigidity, MNm^2		Pile flexural stiffness, MNm^2	Pile Poisson's ratio	Soil Poisson's ratio	Soil friction angle, degrees	Soil buoyant unit weight, kN/m^3
			single pile	pile groups	single pile	pile groups					
Loose sand	10.8	1.2	0.664	0.760	162.0	169.9	220.5	0.25	0.2	33	13.7
Dense sand									0.3	39	14.7

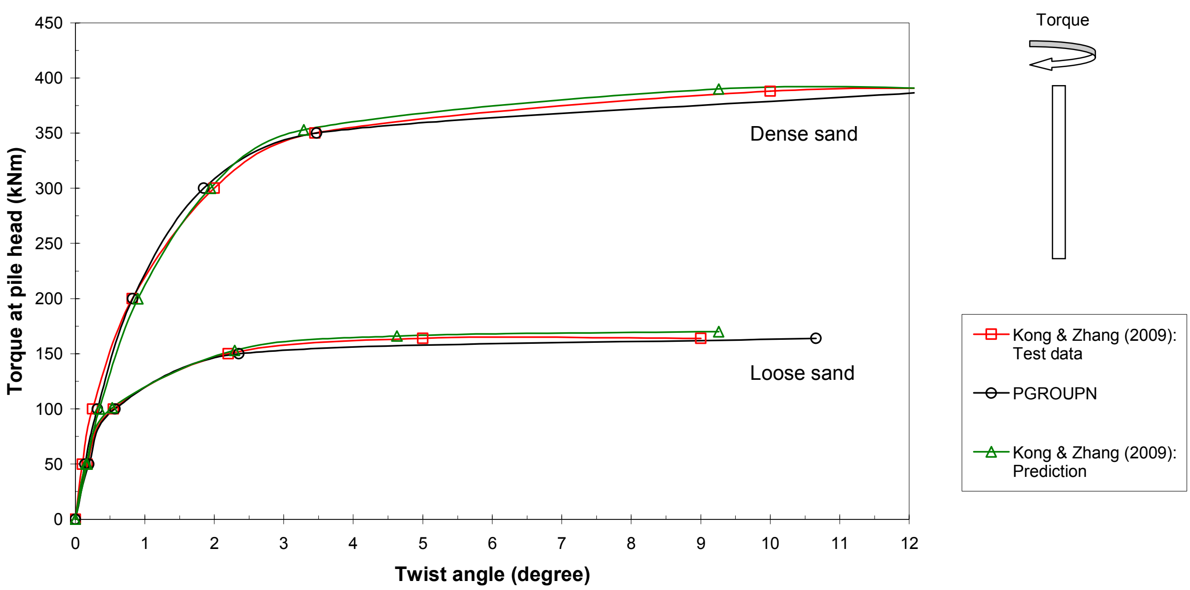

Figure 5: Torque-twist angle curves for single pile

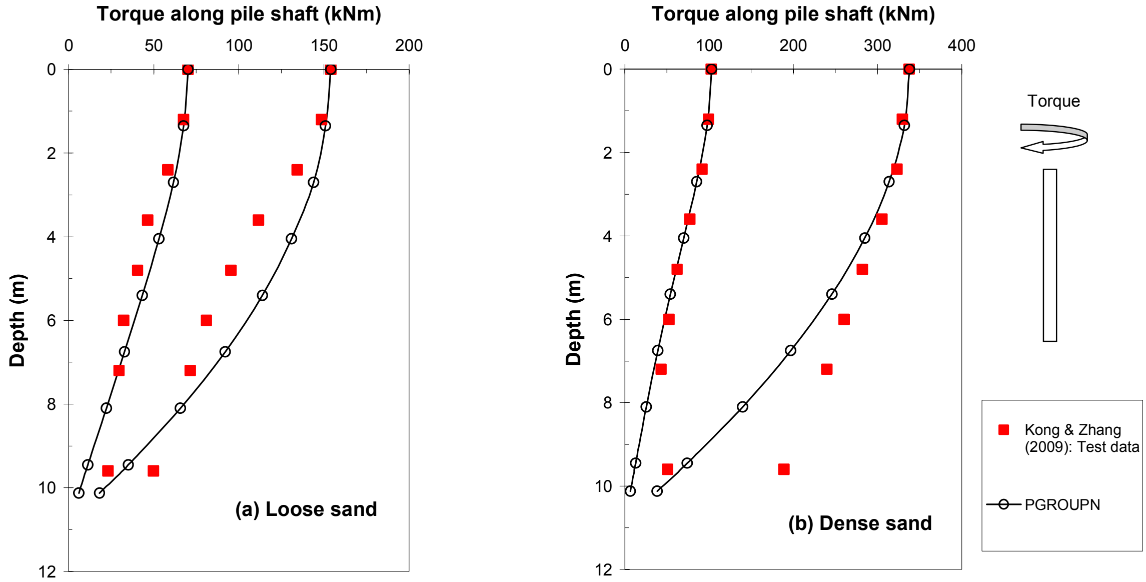

Figure 6: Torque distribution along depth for single pile

It is noted that the above values of soil modulus are relatively low when compared to the values normally adopted in the analysis and design of full-scale piles in sand. However, as reported by Kong (2006), pile jacking in centrifuge tests has significant effects on the soil adjacent to the pile and therefore the value of soil modulus can be significantly different from the value before pile jacking. In addition, scale effects remain an important issue in centrifuge modelling and a higher influence of pile-soil interface properties in a model than in a prototype can be expected. Thus, in the PGROUPN analyses for single pile, the above values of E_s and K_s were selected in order to fit the initial portion and the failure load of the torque-twist angle curves obtained from the centrifuge tests. Then, in the subsequent pile group analyses, the same values of E_s and K_s used for the single-pile analyses have been kept (i.e. without any curve fitting with the test data) in order to simulate the application of PGROUPN in normal design (when only single-pile test data, if any, is available). An instructive application of such design philosophy is described by Hardy & O'Brien (2006) for the design of pile foundations under general loading conditions for the new Wembley Stadium in London.

Turning to pile-group behaviour, the centrifuge tests were carried out on piles arranged in 1x2, 2x2, and 3x3 configurations (*see* inset to Figs. 7-8), with a centre-to-centre spacing of three pile diameters and connected by a rigid cap (1.2m tick) with a clearance of 1.2m from the groundline in prototype. Figures 7 and 8 show the experimental torque-twist angle curves and the corresponding numerical predictions from Kong & Zhang (2009) and from PGROUPN. A good agreement between test data and numerical predictions is observed for the 1x2 and 2x2 pile groups, while the numerical analyses (particularly that from Kong & Zhang) tend to overpredict the pile resistance at high load levels for the 3x3 pile groups. As discussed by Kong (2006), the effects of pile installation may be a possible reason for this discrepancy; namely, pile jacking densifies the soil inside and near the pile groups in loose sand but loosens the soil inside and near the pile groups in dense sand. This effect becomes more pronounced as the number of piles in the group increases.

It should be emphasised that, for both single piles and pile groups, the numerical predictions of Kong & Zhang (2009) are based on a back-analysis of the data from the centrifuge tests, including the use of a back-calculated parameter (i.e. the coupling coefficient, β) in order to reproduce the experimental curves for the pile groups. By contrast, the PGROUPN analyses for the pile groups were carried out using the same soil parameters adopted for the single-pile analyses, without using any additional curve-fitting parameter to improve the agreement with the centrifuge test data. It is noted, however, that the PGROUPN results are of comparable accuracy to those obtained from the numerical predictions of Kong & Zhang, thereby confirming the validity of the proposed PGROUPN approach as a practical tool for pile group design. Figures 7 and 8 also show that inclusion of interaction effects with respect to the effect of torsional movement on the torsional behaviour of adjacent piles leads, in the PGROUPN analysis, to a maximum increase of the twist angle of about 4% for the 3x3 group in dense sand.

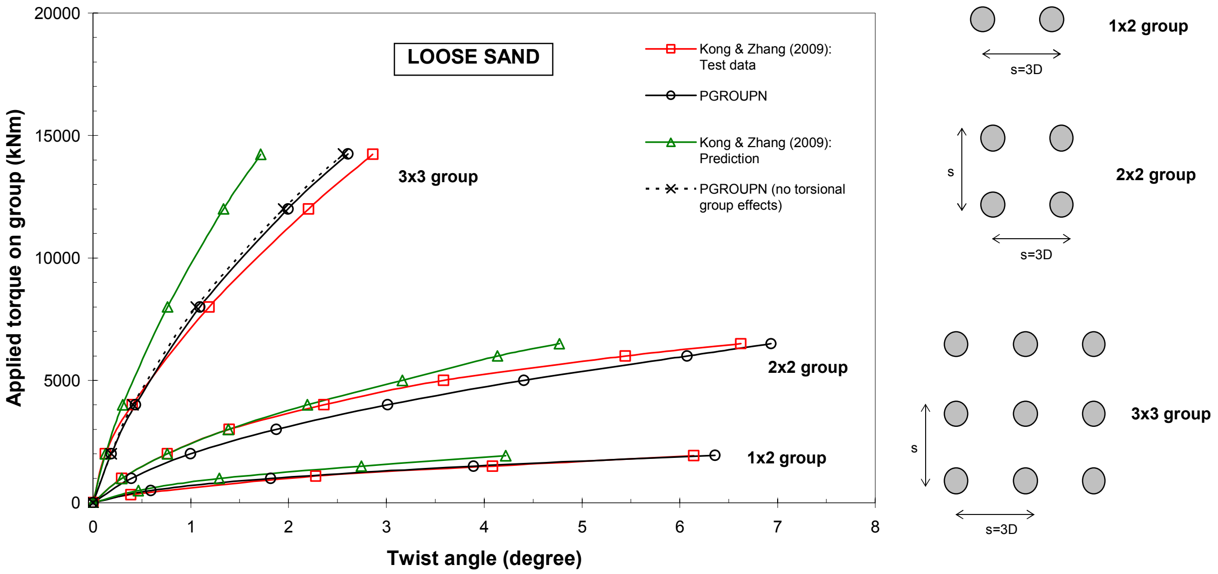

Figure 7: Torque-twist angle curves for pile groups in loose sand

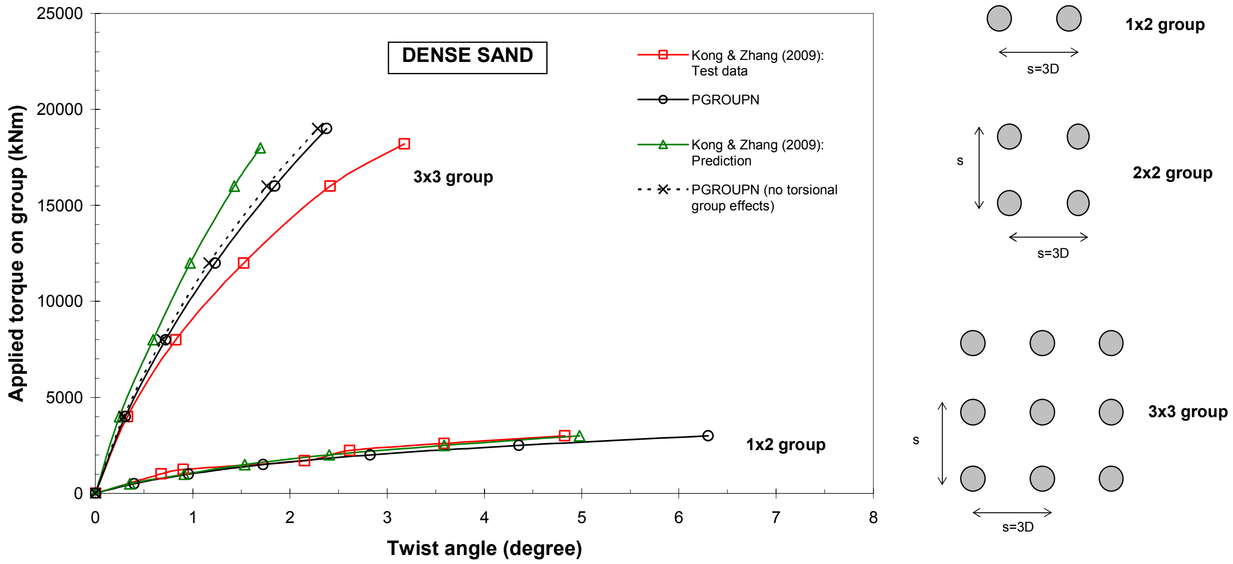

Figure 8: Torque-twist angle curves for pile groups in dense sand

As shown previously in Figure 1, the sustained torque by each pile in the group is shared by the pile torsional component (T_i) plus the lateral contribution from the pile shear force H_iS_i (where H_i is the pile shear force and S_i is the distance of the pile from the torsional centre of the group). Figure 9 shows the decomposition of the torsional resistance components for a pile of the 1x2 group in dense sand, as an example. It is worth noting that the torsional contribution is largely mobilised at a twist angle of about 2.5 degrees, while the lateral contribution continues to increase with the twist angle. This feature of behaviour (also found by Kong & Zhang in the other pile group tests) implies that, at small twist angles, the torsional resistances take larger proportions of the sustained torques and that the proportions decrease at large twist angles. Although some discrepancies (up to about 20%) between the PGROUPN predictions and the test data are observed, it is worth noting that PGROUPN is capable of capturing the above feature of behaviour. For the same 1x2 pile group in dense sand, Figure 10 shows the bending moment and torque profiles for two values of the total applied torque (T = 1019 kNm and T = 2246 kNm). A fair agreement between PGROUPN and test data is found. Finally, it should be observed that instrumentation and measurement errors in the centrifuge test data up to 13% were reported by Kong & Zhang (2007).

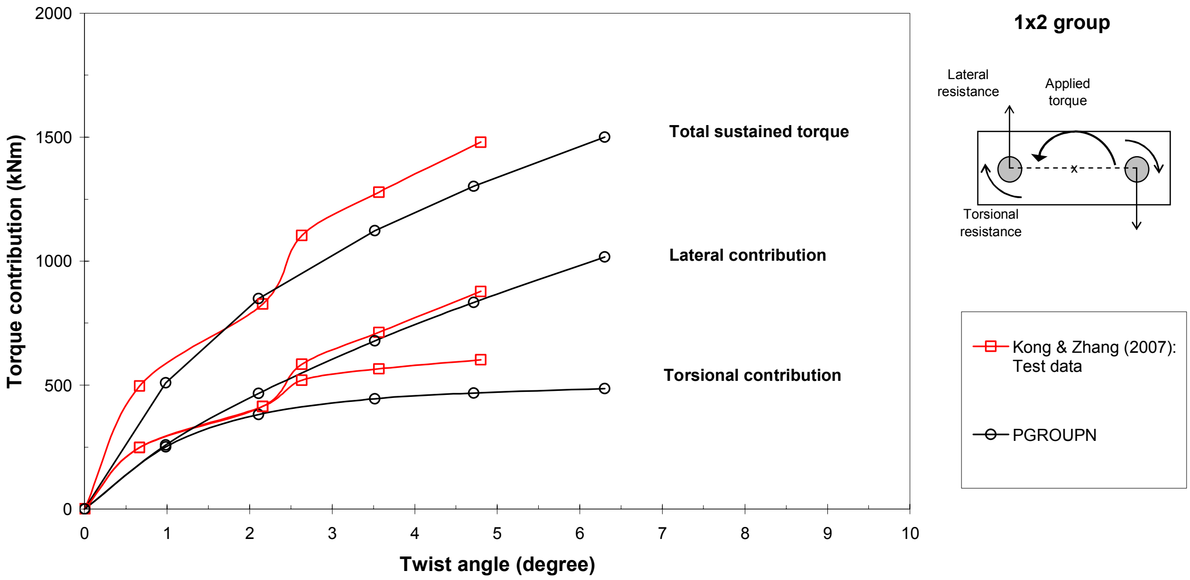

Figure 9: Components of sustained torque in 1x2 pile group in dense sand

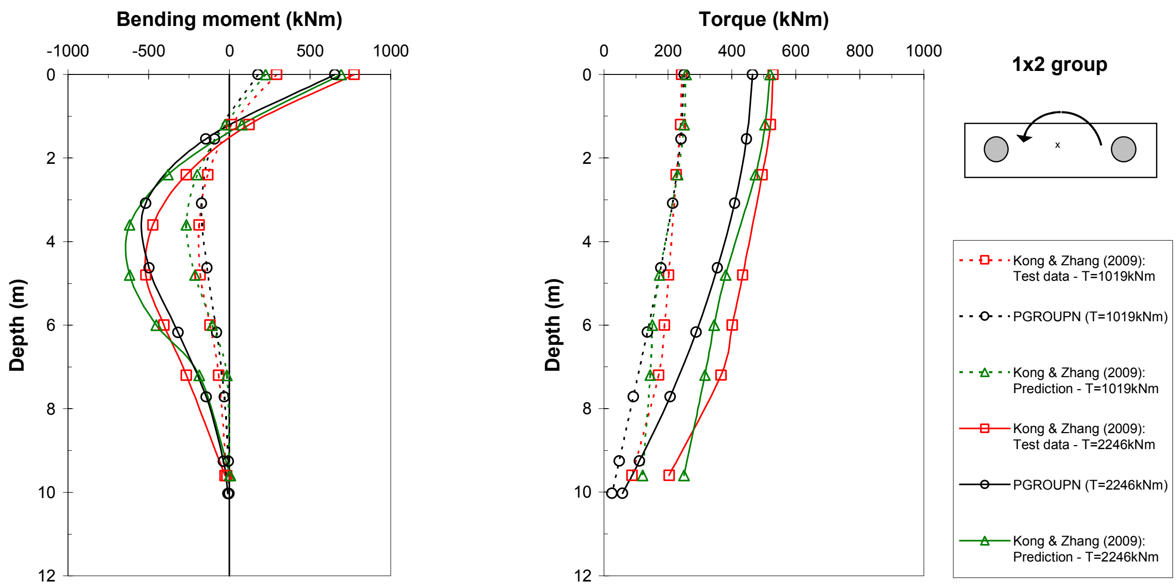

Figure 10: Distribution of bending moment and torque along depth in 1x2 pile group in dense sand

COMPARISON WITH FIELD TESTS BY STOLL (1972)

A further application of PGROUPN is illustrated by the analysis of full-scale torsional load tests reported by Stoll (1972). The tests were conducted on two steel pipe piles, backfilled with concrete, of external diameter 0.273 m, 6.3 mm wall thickness, and a torsional rigidity $(GJ)_p$ of 12.8 MNm^2. The piles were driven into deposits consisting of layers of soft organic silt, overlying clayey silt, sand, and gravel. Pile A-3 was driven to a penetration depth of 17.4 m, while the other pile, pile V-4, was driven to 20.7 m.

Following the SPT *N*–values reported by Stoll, the PGROUPN analyses are based on a subsoil idealisation into two layers, as summarised in Table 3 below. For the top layer of soft clay, a constant value of *N* equal to 4 has been assumed for both piles. The underlying sandy layer has been modelled on the basis of an *N* value of 4 at the top, increasing linearly to 20 at the bottom of the layer, for pile A-3, while values of *N* equal to 5 at the top, increasing linearly to 50 at the bottom of the layer, have been adopted for pile V-4. Based on the above values of *N*, the initial values of soil modulus (E_s) shown in Table 3 have been estimated from the following empirical correlations, generally used for the axial response of piles (refer to Poulos, 1994):

$$E_s = 14N \quad [\text{MPa}] - \text{clay} \qquad \text{(Equ. 7)}$$

$$E_s = 16.9N^{0.9} \quad [\text{MPa}] - \text{sand} \qquad \text{(Equ. 8)}$$

In order to define the limiting pile-soil stresses of Equs. (4)-(5) for the PGROUPN analyses, a value of undrained shear strength (C_u) equal to 18 kPa for the top layer has been estimated from the common correlation C_u = 4.5N (Stroud, 1975), while the friction angles (ϕ) of 30^o and 34^o for the underlying layer have been estimated from the SPT N-values, as described by Peck *et al.* (1967). The values of the coefficient of horizontal soil stress (K_s) have been selected in order to match the ultimate torques measured in the field test, i.e. T_u = 29.3 kNm and T_u = 52.1 kNm for piles A-3 and V-4, respectively. Finally, the hyperbolic curve fitting constant (R_f) for the torsional response has been taken as 0.99.

Figure 11 shows a generally good agreement between PGROUPN and the field measurements for the torque-twist angle curves (where r_o is the pile external radius, equal to 0.1365m), particularly at low load levels. It is of interest to observe that PGROUPN is capable of reproducing with good accuracy the initial portion of the measured curves. This initial portion is mainly governed by the values of soil modulus defined in Equations 7-8, thereby providing further support to the conclusions of Poulos (1975) and Randolph (1981), i.e. the soil modulus adopted for the axial response can be used with reasonable confidence to predict the response of piles subjected to torsion.

Table 3: Input soil parameters adopted for the PGROUPN analyses

	Soil layer 1 (undrained clay)							
	Thickness (m)	Soil modulus, E_s (MPa)	Rate of increase of soil modulus with depth, m_{Es} (MPa/m)	Poisson's ratio, υ_s	Undrained shear strength, C_u (kPa)	Rate of increase of undrained shear strength, m_{Cu} (kPa/m)	Adhesion factor, α	Buoyant unit weight, γ' (kN/m^3)
Pile A-3	5.5	56.0	0.0	0.5	18.0	0.0	0.5	8.0
Pile V-4	3.7							
	Soil layer 2 (drained sand)							
	Thickness (m)	Soil modulus, E_s (MPa)	Rate of increase of soil modulus with depth, m_{Es} (MPa/m)	Poisson's ratio, υ_s	Friction angle, ϕ (degrees)	Interface friction angle, δ (degrees)	Coefficient of horizontal soil stress (K_s)	Buoyant unit weight, γ' (kN/m^3)
Pile A-3	11.9	56.0	16.3	0.3	30.0	0.8ϕ	0.4	8.0
Pile V-4	17.0	72.0	29.4		34.0		0.5	

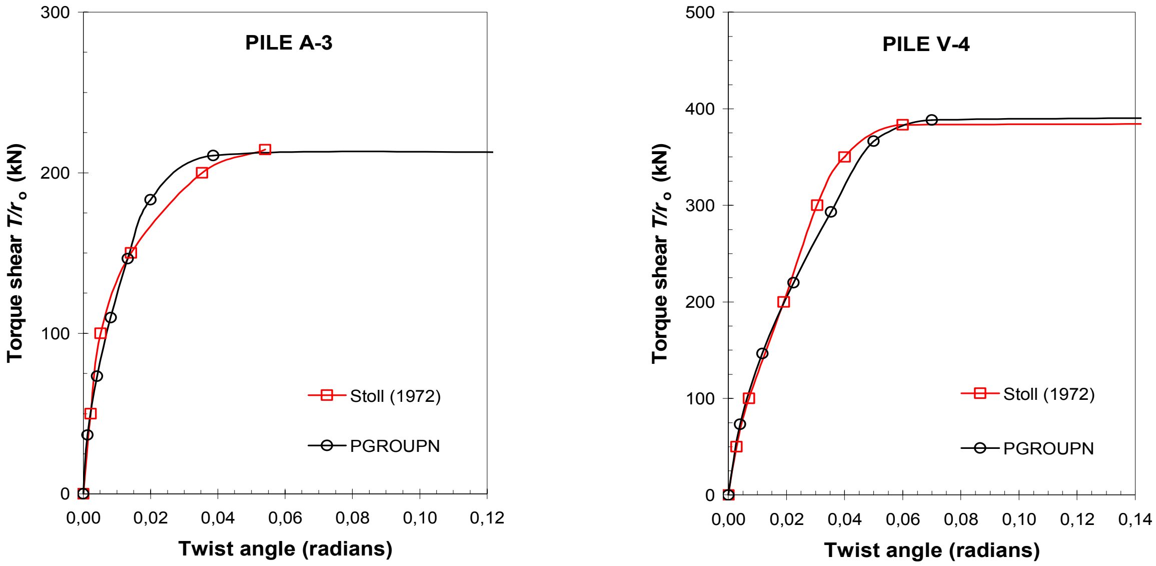

Figure 11: Comparison with torque-twist angle curves measured by Stoll (1972) for single pile

CONCLUSIONS

A numerical method of analysis, based on a complete BEM solution of the soil-continuum and implemented in the computer program PGROUPN, has been introduced for the determination of the non-linear response of single piles and pile groups to torsional loading. The proposed method has been successfully validated through comparison with alternative numerical analyses, analytical solutions, centrifuge model tests and a published case history.

It is found that the simple hyperbolic soil model adopted by PGROUPN is capable of capturing the main non-linear features of pile behaviour under torsion, thereby offering the prospect of more realistic predictions and thus more effective design techniques. Inclusion of torsional interaction effects in a group of piles (i.e. the effects of torsional movement of a pile on the torsional behaviour of adjacent piles) has resulted, within PGROUPN, in a relatively low increase of twist angles (up to a maximum of about 7%). However, the torque distribution in the individual piles is affected by torsional interaction effects, leading to a non-uniform torque distribution among the group piles.

The comparison with the load-transfer analyses (based on *p-y* and τ-θ curves) of Kong & Zhang (2009) has provided further evidence that PGROUPN, by taking into account the continuous nature of pile-soil interaction, removes the uncertainty of empirical load-transfer approaches and provides a simple design tool based on conventional soil parameters. The analysis of the field tests by Stoll (1972) has shown that reasonable predictions of pile response under torque may be obtained using soil parameters derived from the site investigation data, thereby confirming the usefulness of the PGROUPN approach for practical problems, particularly when pile test results are not yet available. It is also found that the values of soil modulus normally adopted for the axial response can be used with sufficient confidence to predict the torsional response of piles.

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